X.1. Section Properties

The following sheet is intended as a demonstration template to show the capabilities and drawbacks of swapping over to Mathcad to perform our calculations. The sections can be identified by typing them in as "section type_size" with no spaces, all section ID's are available in the attached spreadsheet.

$ID := \text{"UB\_203x133x25"}$ $L := 4 \cdot m$ Beam length

$ORIGIN := 1$

	Mass per metre	Depth of section	Width of section	Thick
				Web

Вывод

$ID_e := excel_{\text{"A5:A1357"}}$

$b_e := excel_{\text{"D5:D1357"}} \cdot m$

$d_{fillet\_e} := excel_{\text{"G5:G1357"}} \cdot m$

$c_{YY\_e} := excel_{\text{"J5:J1357"}} \cdot m$

$R_{ZZ\_e} := excel_{\text{"M5:M1357"}} \cdot m$

$W_{pl\_YY\_e} := excel_{\text{"P5:P1357"}} \cdot m^3$

$Root_{radius\_e} := excel_{\text{"S5:S1357"}} \cdot m$

$Mass_{per\_meter\_e} := excel_{\text{"B5:B1357"}} \cdot \frac{kg}{m}$

$t_{web\_e} := excel_{\text{"E5:E1357"}} \cdot m$

$I_{YY\_e} := excel_{\text{"H5:H1357"}} \cdot m^4$

$c_{ZZ\_e} := excel_{\text{"K5:K1357"}} \cdot m$

$W_{el\_YY\_e} := excel_{\text{"N5:N1357"}} \cdot m^3$

$W_{pl\_ZZ\_e} := excel_{\text{"Q5:Q1357"}} \cdot m^3$

$A_{v\_e} := excel_{\text{"T5:T1357"}} \cdot m^2$

$h_e := excel_{\text{"C5:C1357"}} \cdot m$

$t_{flange\_e} := excel_{\text{"F5:F1357"}} \cdot m$

$I_{ZZ\_e} := excel_{\text{"I5:I1357"}} \cdot m^4$

$R_{YY\_e} := excel_{\text{"L5:L1357"}} \cdot m$

$W_{el\_ZZ\_e} := excel_{\text{"O5:O1357"}} \cdot m^3$

$A_{section\_e} := excel_{\text{"R5:R1357"}} \cdot m^2$

$idx := \text{match}(ID, ID_e)_1 = 103$

$h := h_{e_{idx}} = 203.2\ mm$

$b := b_{e_{idx}} = 133.2\ mm$

$t_{web} := t_{web\_e_{idx}} = 5.7\ mm$

$t_{flange} := t_{flange\_e_{idx}} = 7.8\ mm$

$d_{fillet} := d_{fillet\_e_{idx}} = 172.4\ mm$

$Mass_{per\_meter} := Mass_{per\_meter\_e_{idx}} = 25.1\ \frac{kg}{m}$

$Root_{radius} := Root_{radius\_e_{idx}} = 7.6\ mm$

$A_{section} := A_{section\_e_{idx}} = (3.2 \cdot 10^3)\ mm^2$

$A_v := A_{v\_e_{idx}} = (1.285 \cdot 10^3)\ mm^2$

MAJOR AXIS PROPERTIES

$I_{YY} := I_{YY\_e_{idx}} = (23.4 \cdot 10^6)\ mm^4$

$c_{YY} := c_{YY\_e_{idx}} = 101.6\ mm$

$R_{YY} := R_{YY\_e_{idx}} = 85.6\ mm$

$W_{el\_YY} := W_{el\_YY\_e_{idx}} = (230 \cdot 10^3)\ mm^3$

$W_{pl\_YY} := W_{pl\_YY\_e_{idx}} = (258 \cdot 10^3)\ mm^3$

MINOR AXIS PROPERTIES

$I_{ZZ} := I_{ZZ\_e_{idx}} = (3.08 \cdot 10^6)\ mm^4$

$c_{ZZ} := c_{ZZ\_e_{idx}} = 66.6\ mm$

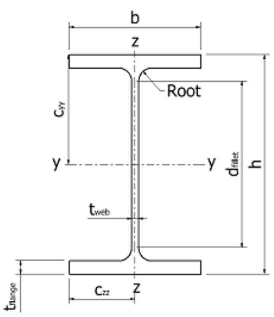
$R_{ZZ} := R_{ZZ\_e_{idx}} = 31\ mm$

$W_{el\_ZZ} := W_{el\_ZZ\_e_{idx}} = (46.2 \cdot 10^3)\ mm^3$

$W_{pl\_ZZ} := W_{pl\_ZZ\_e_{idx}} = (70.9 \cdot 10^3)\ mm^3$

Material Properties

Universal beam (UB)



$E := 210 \cdot GPa$

$f_y := 275 \cdot MPa$

$UTS := 430\ MPa$

$\varepsilon := \sqrt{\frac{235 \cdot MPa}{f_y}} = 0.92$

If using materials with a thickness greater than 16 mm a reduced yield strength should be used. As per BS EN 10025-2.

Note: Double click section picture to select appropriate image.

Section Geometric Properties

$h = 203.2\ mm$ $\quad d_{fillet} = 172.4\ mm$ $\quad c_{YY} = 101.6\ mm$ $\quad L = 4\ m$

$b = 133.2\ mm$ $\quad Root_{radius} = 7.6\ mm$ $\quad c_{ZZ} = 66.6\ mm$

$t_{web} = 5.7\ mm$

$t_{flange} = 7.8\ mm$ $\quad A_{section} = 3200\ mm^2$

Derived Section Properties

Second Moment of Area

$I_{YY} = \left(23.4 \cdot 10^6\right)\ mm^4$

$I_{ZZ} = \left(3.08 \cdot 10^6\right)\ mm^4$

Elastic Modulus

$W_{el\_YY} = \left(230 \cdot 10^3\right)\ mm^3$

$W_{el\_ZZ} = \left(46.2 \cdot 10^3\right)\ mm^3$

Plastic Modulus

$W_{pl\_YY} = \left(258 \cdot 10^3\right)\ mm^3$

$W_{pl\_ZZ} = \left(70.9 \cdot 10^3\right)\ mm^3$

Radius of Gyration

$R_{YY} = 85.6\ mm$

$R_{ZZ} = 31\ mm$

Shear Area

$A_v = \left(1.285 \cdot 10^3\right)\ mm^2$

$Mass_{per\_meter} = 25.1\ \frac{kg}{m}$

Section Classification

As per BS EN 1993-1-1:2005, the section should be classified according to table 5.2. The results from the flange classification will determine which section modulus to use for the moment resistance of the section.

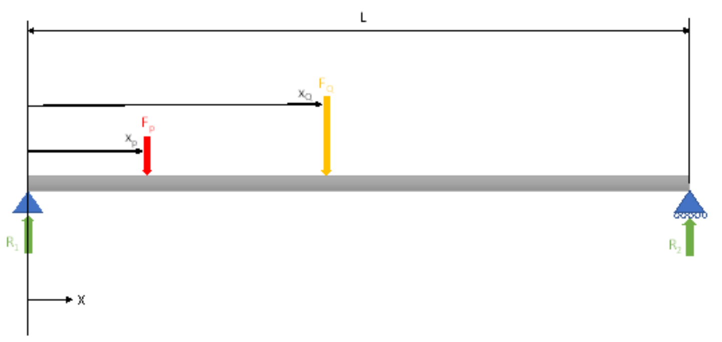
Web Classification:

Flange Classification:

X.2. Loading Arrangement

X.2.1: Loading Arrangement

In the following section permanent actions on the beam shall be denoted by the subscript G and variable actions on the beam shall be denoted by the subscript Q. UDLs applied over the entire beam length can be entered in the appropriate UDL box. For a conservative assessment UDLs acting over a partial section of the beam can be represented by an equivalent point loading.



Note: The loading arrangement diagram can be altered by double clicking. This will open the power point.

$\gamma_G := 1.35$ Load factor for permanent actions. $\gamma_Q := 1.5$ Load factor for variable actions. $\xi := 0.925$ Reduction factor (permanent).

UDLs

$UDL_{self} := Mass_{per\_meter} \cdot g = 0.246 \frac{kN}{m}$

$UDL_g := 0 \frac{kN}{m}$ Permanent UDL.

$UDL_q := 0 \frac{kN}{m}$ Variable UDL.

Point Loads

$F_G := \begin{bmatrix} 6 \\ 8 \end{bmatrix} \cdot kN$

$F_Q := [4] \cdot kN$

Distance relative to R1

$x_G := \begin{bmatrix} 1.3 \\ 2.8 \end{bmatrix} \cdot m$ Permanent

$x_Q := [0.8] \cdot m$ Variable

The following collapsible area shows the equations for determination of shear force and moment equations for the beam.

Define index i and j for shear point load permanent and variable vectors:

$ni := \text{rows}(F_G)$ $nj := \text{rows}(F_Q)$

$i := 1..ni$ $j := 1..nj$

Shear force equation for permanent actions:

$$V_G(x) := \begin{array}{l} \text{for } i \in 1..ni \\ \quad V_{G_i} \leftarrow \text{if}\left(x < x_{G_i}, \frac{L - x_{G_i}}{L}, \frac{-x_{G_i}}{L}\right) \cdot \gamma_G \cdot F_{G_i} \\ V_G \end{array}$$

Shear force equation for variable actions:

$$V_Q(x) := \begin{array}{l} \text{for } j \in 1..nj \\ \quad V_{Q_j} \leftarrow \text{if}\left(x < x_{Q_j}, \frac{L - x_{Q_j}}{L}, -\frac{x_{Q_j}}{L}\right) \cdot \gamma_Q \cdot F_{Q_j} \\ V_Q \end{array}$$

Determine the combined distributed loading:

$$w_u := UDL_{self} + UDL_g \cdot \gamma_G + UDL_q \cdot \gamma_Q$$

Shear force equation for combined UDLs:

$$V_w(x) := \left(\frac{L}{2} - x\right) \cdot w_u$$

Total shear force is summation of shear force:

$$V_{total}(x) := V_w(x) + \sum_i \left(V_G(x)_i\right) + \sum_j \left(V_Q(x)_j\right)$$

Reaction forces:

$R1 := V_{total}(0\ m) = 14000\ N$

$R2 := V_{total}(L) = -11885\ N$

Design shear

$V_{ED} := \max(R1, R2) = 14000\ N$

Distributed moments:

$$M_{self}(x) := (L - x) \cdot \frac{UDL_{self} \cdot x}{2} \qquad M_g(x) := (L - x) \cdot \frac{UDL_g \cdot x \cdot \gamma_G}{2} \qquad M_q(x) := (L - x) \cdot \frac{UDL_q \cdot x \cdot \gamma_Q}{2}$$

Moment equation for permanent actions:

$$M_G(x) := \begin{array}{l} \text{for } i \in 1..ni \\ \quad M_{G_i} \leftarrow \text{if}\left(x < x_{G_i}, \frac{L - x_{G_i}}{L} \cdot x, \frac{x_{G_i}}{L} \cdot (L - x)\right) \cdot \gamma_G \cdot F_{G_i} \\ M_G \end{array}$$

Moment equation for variable actions:

$$M_Q(x) := \left\| \begin{array}{l} \text{for } j \in 1..nj \\ \quad \left\| M_{Q_j} \leftarrow \textbf{if}\left(x < x_{Q_j}, \frac{L - x_{Q_j}}{L} \cdot x, \frac{x_{Q_j}}{L} \cdot (L - x)\right) \cdot \gamma_Q \cdot F_{Q_j} \right. \\ M_Q \end{array} \right.$$

Total bending moment is summation of moments:

$$M_{total}(x) := M_{self}(x) + M_g(x) + M_q(x) + \sum_i \left(M_G(x)_i\right) + \sum_j \left(M_Q(x)_j\right)$$

Split beam into k-nodes::

$n := 500 \quad k := 1..n+1$

$$M_k := M_{total}\left(\frac{L}{n} \cdot (k-1)\right) \quad x_p := 0, \frac{L}{n}..L$$

Design moment

$M_{Ed} := \max(M) = 14990\ \mathbf{N \cdot m}$

The deflections in the beam can then be determined from conjugate beam theory for each of the loading types.

Permanent Loading:

$$F_{conj\_G} := \int_0^L M_{self}(x) + M_g(x) + \sum_i M_G(x)_i \, dx \qquad Fxx_{conj\_G}(xx) := \int_0^{xx} M_{self}(x) + M_g(x) + \sum_i M_G(x)_i \, dx$$

$$x_{conj\_G} := \frac{\int_0^L M_{self}(x) \cdot x + M_g(x) \cdot x + \left(\sum_i M_G(x)_i\right) \cdot x \, dx}{F_{conj\_G}}$$

$$xx_{conj\_G}(xx) := \frac{\int_0^{xx} M_{self}(x) \cdot x + M_g(x) \cdot x + \left(\sum_i M_G(x)_i\right) \cdot x \, dx}{Fxx_{conj\_G}(xx)}$$

$$R_{conj\_G} := F_{conj\_G} \cdot \frac{\left(L - x_{conj\_G}\right)}{L}$$

$$\delta_G(xx) := R_{conj\_G} \cdot xx - Fxx_{conj\_G}(xx) \cdot \left(xx - xx_{conj\_G}(xx)\right)$$

Variable Loading:

$$F_{conj\_Q} := \int_0^L M_q(x) + \sum_j M_Q(x)_j \, dx \qquad Fxx_{conj\_Q}(xx) := \int_0^{xx} M_q(x) + \sum_j M_Q(x)_j \, dx$$

$$x_{conj\_Q} := \frac{\int_0^L M_q(x) \cdot x + \left(\sum_j M_Q(x)_j\right) \cdot x \, dx}{F_{conj\_Q}} \qquad xx_{conj\_Q}(xx) := \frac{\int_0^{xx} M_q(x) \cdot x + \left(\sum_j M_Q(x)_j\right) \cdot x \, dx}{Fxx_{conj\_Q}(xx)}$$

$$R_{conj\_Q} := F_{conj\_Q} \cdot \frac{\left(L - x_{conj\_Q}\right)}{L}$$

$\delta_Q(xx) := R_{conj\_Q} \cdot xx - Fxx_{conj\_Q}(xx) \cdot (xx - xx_{conj\_Q}(xx))$

Total deflection:

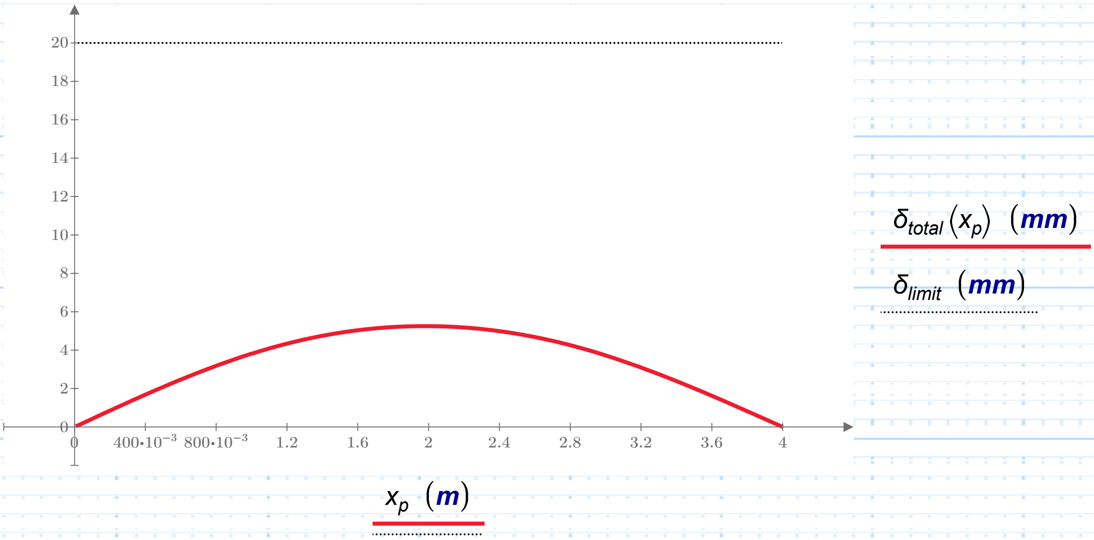
$n := 500$ $\quad k := 1..n+1$ $\quad x\delta_k := (k-1) \cdot \left(\frac{L}{n}\right)$ $\quad x\delta_1 := 0.00000001\ m$

$\delta_{total}(xx) := \frac{1}{E \cdot I_{YY}} (\delta_G(xx) + \delta_Q(xx))$

$\delta_k := \delta_{total}(x\delta_k)$ $\quad \delta_{max} := \max(\delta) = 5.254\ mm$

$k_{match\_x} := \mathrm{match}(\delta_{max}, \delta)$

$\delta_{limit} := \frac{L}{200}$



$$\delta = \begin{bmatrix} \vdots \\ 5.147 \\ 5.154 \\ 5.161 \\ 5.166 \\ 5.172 \\ 5.177 \\ 5.182 \\ 5.188 \\ 5.192 \\ 5.197 \\ 5.202 \\ 5.206 \\ 5.21 \\ 5.214 \\ 5.218 \\ 5.222 \\ 5.225 \end{bmatrix} mm$$

$$\begin{bmatrix} 5.229 \\ 5.232 \\ 5.234 \\ 5.238 \\ 5.24 \\ 5.242 \\ 5.244 \\ 5.244 \\ \vdots \end{bmatrix}$$