Fatigue strengths thickness dependence in welded constructions

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Abstract

In this thesis work the thickness effect of cruciform specimens with non-load carrying welds has been investigated.

It is well known from both fatigue testing and theoretical analysis based on fracture mechanics, that increasing the thickness of welded members while maintaining all the other parameters will, in general, cause a decrease in fatigue strength. The aim here was to investigate the reversed thickness effect i.e. if fatigue strength increases with decreasing thickness under a limit value t_0. Attempts to extrapolate the thickness correction formula below the limit value t_0 and also calculate the thickness correction exponent n were made.

To investigate this the following were performed:

1. A literature survey on important standards, recommendations and pertinent articles.

2. Fatigue testing of 6 and 12 mm non-load carrying transverse fillet weld (cruciform) joint in pure tension and pure bending. The result of the testing show that the fatigue life were approximately twice as high for 6 mm in bending than for the 12 mm. An attempt to calculate the thickness correction exponent resulted in n = 0,32 in bending.

3. Facture mechanics analyses were performed using FEM and according to the requirements in the British Standard. The FEM analyses indicated that the fatigue life for the 6 mm specimen was about twice that of the 12 mm specimen. Attempts were also made to calculate the thickness correction exponent resulting in n = 0,26 (bending) and n = 0,31 (tension).

Sammanfattning

I detta examensarbete har utmattningshållfasthetens tjockleksberoende på korsprovstavar med icke lastbärande svetsar undersökts.

Det är väl känt och dokumenterat från både utmattningsprovning och teoretiska analyser baserade på brottmekanik, att om tjockleken ökar över en viss gränstjocklek t_0 medan alla andra parametrar förblir oförändrade på svetsade detaljer sjunker utmattningshållfastheten generellt. Målet med detta examensarbete var att undersöka den omvända tjocklekseffekten, det vill säga om utmattningshållfastheten ökar med sjunkande tjocklek under gränsvärdet t_0.

Tjockleks korrektionsexponenten *n* i tjocklekskorrektions formeln beräknades även.

För att undersöka detta utfördes följande:

1. En litteraturstudie som sammanfattade viktiga standarder, rekommendationer och artiklar.

2. Utmattningsprovning av 6 och 12 mm korsprovstavar med icke lastbärande svetsar i drag och böj belastning. Resultaten av provningen visar att livslängden var ungefär två gånger längre för 6 mm provstavarna än 12 mm provstavarna i böjbelastning. Tjocklekskorrektionsexponenten beräknades till $n = 0{,}32$ i böjning.

3. Brottmekaniska analyser med hjälp av FEM och British Standard utfördes. FEM analyserna indikerade att livslängden för 6 mm provstavarna var ungefär dubbelt så lång som för 12 mm provstavarna både i dragbelastning och böjbelastning. Tjocklekskorrektionsexponenten beräknades till $n = 0{,}26$ i böjning och $n = 0{,}31$ i drag.

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1 Introduction

This thesis work named ”Fatigue strengths thickness dependence in welded constructions” was performed at SSAB Tunnplåt in Borlänge during the summer and the autumn of 2001.

The thickness dependence in thin welded joints will be investigated according to the scheme.

1. Literature survey
2. Fatigue testing of 6 and 12 mm joints
3. Fracture mechanics analysis to confirm the test results

An estimation of the theoretical and experimental thickness effect exponent will be made.

2 Definition of the problem

In this chapter the problem will be defined.

2.1 Background

The fatigue strength thickness dependence in welded constructions has previously been studied.

The thickness effect originate from three different causes,

1. Statistical effects
2. Technological effects
3. Stress gradient effect.

The thickness dependence is today used in many norms and recommendations e.g. BSK (The Swedish national board of housing, building and planning handbook of steel constructions), BS (British Standard) and SSAB Sheet steel handbook. All these recommendations are based on fatigue testing.

Typically tests have been performed on relatively thick specimens, since it has been interesting to evaluate how the fatigue strength decreases with increasing thickness, especially for the offshore industry. The outcome of these testing usually is that the fatigue strength decreases with increasing thickness over a limit value t_0 where t_0 usually varies between 15 - 30 mm. If the fatigue strength increases with decreased thickness below the limit value t_0 is not thoroughly examined. Some recommendations like SSAB sheet steel handbook just extrapolate the values achieved from testing on thicker specimens to thinner specimens.

2.2 Method

This study is based on three parts: literature survey, testing and fracture mechanics analysis. This will be outlined next.

2.2.1 Literature survey

It is important to compile some important standards and recommendations regarding the thickness effect. Therefore this literature study based on both standards and published articles will be put together.

2.2.2 Testing

To examine the thickness effects on thin structures. Fatigue testing was performed on non-load carrying transverse fillet welded cruciform joints of the thicknesses 6 and 12 mm in both pure tension and pure three points bending, respectively. The joints were made of the steel DOMEX 550 MC, produced by SSAB Tunnplåt. The testing was carried out by SSAB Tunnplåt's laboratory in Borlänge Sweden.

If a significant difference in fatigue life between the two thicknesses are found the constant n in the thickness effect law can be calculated

$$s = s_0 \cdot \left(\frac{t_0}{t} \right)^n . \qquad (1)$$

This law will be further explained in the literature survey, but in short it is used to calculate the fatigue strength s, which is reduced because of the thickness effect.

2.2.3 Fracture mechanics analysis

As a complement to the testing, fracture mechanics calculations were made in an attempt to evaluate the effect of the stress concentration thickness dependence.

Finding the geometry dependence

In order to isolate the geometric part of the thickness effect and validate the testing a fracture mechanics analysis were carried out.

An often-used formula to calculate the stress intensity factor K_I is

$$K_I = s \cdot \sqrt{p \cdot a} \cdot f \,, \qquad (2)$$

where s is the stress applied and a is the crack depth. The factor f accounts for effects pertaining to a specific geometry.

The stress intensity factor K_I characterizes the influence of load on the crack-tip stress and strain fields and is a measure of the tendency for crack propagation i.e. the crack driving force. In this case the K_I factor will be achieved by using finite element analyses (FEM) and the geometry factor f will be computed as

$$f = \frac{K_I}{s \cdot \sqrt{p \cdot a}} \,. \qquad (3)$$

Calculation of fatigue life

It is also of interest to calculate the fatigue life of the specimens, by use of Paris crack growth law. The number of cycles to failure can be calculated by

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C \cdot ?\, K_I{}^m \,, \qquad (4)$$

where $\mathrm{d}a/\mathrm{d}N$ is the crack growth rate per cycle and the C and m are material constants. The stress intensity factors K_I will here be calculated from a finite element analysis.

Paris law can be used when it is known that the welding process introduces inherent surface crack-like flaws at the weld toe, i.e., along the fusion line. These flaws are regarded as initiated cracks, which is a somewhat conservative assumption.

When the fatigue life is calculated it will be possible to extract n in the thickness effect law (1). The fatigue life calculations based on FEM were compared with the real test values. In addition, a fracture mechanics analysis based on an accepted standard was done as a reference. The standard chosen for this was the British Standard BS 7910:1999 [13].

3 Literature survey

In this chapter a summary of important standards, recommendations and articles regarding the thickness effect will be made.

3.1 Descriptions

3.1.1 Three main explanations for the thickness effect

According to Örjasäter [1], the thickness effect could be explained by the following three points:

Statistical size effect

There is an increased likelihood of finding a significant large defect in a larger volume than in a small volume. Since fatigue is a weakest link process, a decreasing fatigue life with increasing specimen size can be expected [1].

The technological size effect

In welded joints the weld toe radius is fairly constant and independent of plate thickness. Therefore the stress concentration factor will increase with increasing plate thickness. This will lead to a thickness effect and according to [1] this is the main contribution to the thickness effect. Örjasäter also presents a diagram where the stress concentration at different thickness with constant weld toe radius is presented, which shows that the stress concentration increases from approximately 2 at 50 mm up to 2,9 at 150 mm thickness. See also Figure 2 for an explanation of where the weld toe radius is.

If the material is surface treated, e.g. by cold working or by induction hardening, the depth of the surface treatment is usually independent of the component thickness. This will lead to a decrease in fatigue strength with increased component thickness.

There is also a difference in deformation and in temperature history between thick and thin specimens.

Stress gradient effect

Stress gradients are caused by geometrical discontinuities, by bending or torsional loads. The reason for the stress gradient effect is that a crack at the surface of a thick specimen will experience a larger stress during crack initiation and early crack propagation than a crack of the same length in a thin specimen. The difference in initial crack growth rate overmatches the difference in crack length to cause fracture; hence the thinner joint will have a longer fatigue life. Figure 1 describes the difference in the stress gradient between two different thicknesses.

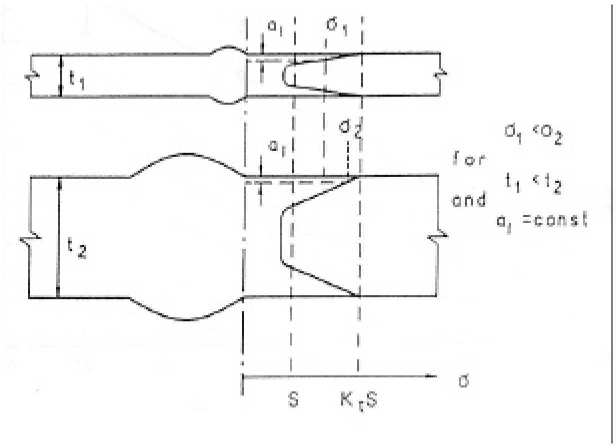

Figure 1 A simple model to describe the stress gradient effect [1].

According to Örjasäter [1] the two major factors influencing the thickness effect are the technological size effect and the statistical size effect. But, Niemi [2] states that the geometrical effect, i.e. technological size effect and stress gradient effect, alone often dominates the contribution to the thickness effect.

3.1.2 Influence of the weld geometry

Weld toe

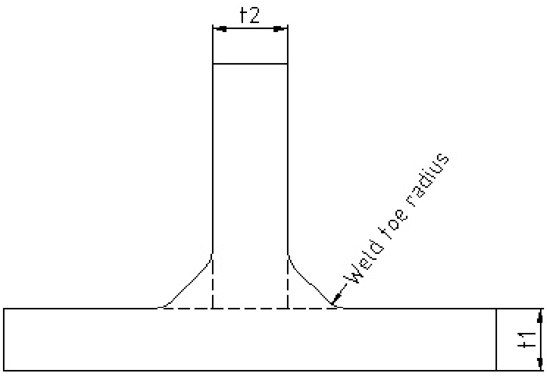

Figure 2 Drawing showing the location of the weld toe.

The stress intensity factor will vary with the radius of the weld toe. If the radius increases the stress intensity will decrease. The notch root radius at the weld toe is often only dependent of the last pass at the weld toe [3]. Therefore the radius is often independent of the plate thickness. This implies that the local stress concentration is dependent of plate thickness. If the weld toe is being improved with e.g. grinding so that the stress concentration factor is reduced the thickness effect might be reduced.

Effect of weld angle

D. Bowness [4] has investigated different weld angles (30-75°) in T-butt joints and came to the conclusion that there were a difference in the weld toe magnification factor, *Mk*. The effect of the weld angle is greater on shallower cracks (a/t<0,04) than on deeper cracks, and the increase in *Mk* for shallower cracks is more pronounced for bending loading. And these effects are a result of the increased stress raising effect of steeper weld angles.

Influence of attachments
If the plate thickness t_1 and attachment thickness t_2 are different the thickness effect will vary see Figure 2. If $t_2 << t_1$ and the attachment is transverse to the loading direction fracture mechanics based life predictions has shown a significantly lower thickness effect [1].

Misalignment
The methods normally used to calculate the thickness effect doesn't consider the effect of misalignments as discussed by Fricke [5].

When the thickness becomes larger, especially in plate structures, the effect of the misalignment, both angular and axial, will be reduced. Thus there exists a reversed thickness effect. Fricke [5] speculates that this reversed thickness effect might be responsible for the existence of the reference thickness t_0. Below t_0 the thickness effect and the reversed thickness effect approximately is cancelling each other [5].

Other
According to Örjasäter [1] there is no significant difference in the thickness effect between AW (as welded) and PWHT (post weld heat treated) welds.

It is also stated that the thickness effect seems to be smaller for welded joints tested in axial loading than for welded joints in bending.

Possible explanations for this are:

- An initially semicircular crack front has a stronger tendency to form a straight front in bending than for axial loading and a crack with straight front has a tendency to grow faster.

- Furthermore the stress distribution is different. In thick plates the steeper gradient gives stronger thickness effect.

3.1.3 Effects of decreasing thicknesses

The fact that it might be an increase in the fatigue strength for thicknesses below t_0 is not considered in standards and recommendations. But some articles discuss the subject.

Gurney [6] investigate the possibilities to extrapolate the thickness effect design rule (1) below the reference thickness t_0 with different attachment variations. He refers to this as the thinness effect, and reaches the following conclusions:

For axially loaded joints with transverse attachments he confirms that the current "thickness effect" rule (1) could be extrapolated back to thinner joints, down to at least 2 mm for short attachments and 6 mm for longer attachments. But further work is required to confirm whether the limit for longer attachments could be reduced below 6 mm.

The “thinness effect” factor can also be applied to transverse load-carrying fillet welded joints. However, in such joints it is virtually impossible to avoid misalignment. Therefore, in addition, it is important, to build in some level of “expected misalignment” into the design calculations for such joints. Gurney [6] discuss that 1 mm assumed misalignment might be an appropriate assumption.

The fatigue strength of specimens with longitudinal non-load-carrying attachments tended to decrease, rather than increase, as thickness decreased. There is a need for further tests at thin thicknesses to determine whether a reduction in design stress ought to be made for such joints.

Fricke [5] indicates that there is no fatigue strength improvement for thicknesses under t_0 as mentioned above (chapter 3.1.2).

The SSAB Sheet steel handbook [7] recommend that it is always conservative to put $t = t_0$ if $t < t_0$, but also attach a table were the thickness formula (1) extrapolates down to t = 1 – 4 mm. For unwelded materials and spot-welds t_0 / t is set to 1.

3.2 Standards and recommendations

The thickness effect is treated in many standards and recommendations but none of them take into account that there might exist a reversed thickness effect that increases the fatigue strength with decreased thickness.

The International Institute Of Welding

According to the IIW recommendations [8] the reduced strength is taken into consideration by multiplying the fatigue class of the structural detail by the thickness reduction factor

$$f_{red}(t) = \left(\frac{25}{t_{eff}}\right)^n \quad t \geq 25 \text{ mm.} \qquad (5)$$

According to Fricke [5] it is under discussion in IIW if they should recommend the reduced reference value to t_0 = 16 mm instead of the 25 mm value used now.

The thickness correction exponent n depends of the effective thickness t_{eff} and the joint category, se table below:

Joint category	Condition	*n*
Cruciform joints, transverse T-joints, plates with transverse attachments	as-welded	0.3
Cruciform joints, transverse T-joints, plates with transverse attachments	toe ground	0.2
Transverse butt welds	as-welded	0.2
Butt welds ground flush, base material, longitudinal welds or attachment	any	0.1

Table 1 The thickness correction exponent *n* for different weld types.

The plate thickness correction factor is not required in the case of assessment based on effective notch stress procedure or fracture mechanics. See Hobbacher [8] for explanation of the *L* and *t* values.

If $\frac{L}{t} \leq 2$ then $t_{eff} = 0.5 \cdot L$
Else $t_{eff} = t$

BSK 99
In BSK 99 [9] the characteristic fatigue strength is

$$f_{rk} = C \cdot \left(\frac{2 \cdot 10^6}{n_t} \right)^{\frac{1}{3}} \quad , \tag{6}$$

where *C* is set by the joint class that are the characteristic fatigue strength at $2 \cdot 10^6$ stress alternations with constant tension span. Joint classes for base material and the welded joint is to be found in BSK 99 [9]. n_t is the amount of stress alternations during the constructions supposed life time.

The characteristic fatigue strength f_{rk} is multiplied with a thickness factor

$$j_{\text{dim}} = \left(\frac{25}{t} \right)^{0,0763} , \tag{7}$$

where *t* [mm] is the thickest part of the connecting plate in the dimensioning point.

In BSK99 [9] the reversed thickness effect in materials thinner then the reference thickness t_0 is not considered. Therefore if $t < 25$ mm the j_{dim} factor should be set to1,0.

Eurocode 3
In chapter 9.7.2 in the Eurocode standard [10] the reduction of fatigue strength with increased thickness is considered with the formula

$$\Delta s_{R.t} = \Delta s_R \cdot \left(\frac{25}{t} \right)^{0,25} , \tag{8}$$

where $t > 25$ mm, Δs_R is the fatigue strength and Δs_{Rt} is the fatigue strength of the joint taking the thickness effect into account.

If the actual thickness is thinner than the reference thickness 25 mm the fatigue strength shall be taken as that for a thickness of 25 mm.

This reduction for thickness shall be applied only to structural details with welds transverse to the direction of the normal stresses.

BS 7608:1993 [11]
The fatigue strength of welded joints and of bolts is to some extent dependent on material thickness, strength decreasing with increasing thickness. The correction on stress range is of the following form,

$$S = S_B \cdot \left(\frac{t_B}{t} \right)^{\frac{1}{4}} \tag{9}$$

Where:

S is the fatigue strength of the joint under consideration;

S_B is the fatigue strength of the joint using the basic S_r-N curve;

t is the greater of 16 mm or the actual thickness of the member or bolt diameter under consideration;

t_B is the maximum thickness relevant to the basic S_r-N curve (i.e. t_B = 16 mm for welded joints or 25 mm for bolt diameters).

No thickness correction need to be applied in the case of butt welds with the weld reinforcement machined flush, e.g. joint types 4.1, 6.1 and 7.1, see BS 7608:1993 [11].

British Standard does not allow to calculate any fatigue strength improvement for thicknesses under the limit value t_B.

SSAB Sheet steel handbook
The thickness effect factor

$$j_t = \left(\frac{t_0}{t} \right)^{a} . \tag{10}$$

The characteristic fatigue strength f_{rk0} is multiplied with j_t to achieve the reduced characteristic fatigue strength f_{rk}. t_0 has been chosen to 15 mm and the a factor depends on the type of weld. The a-value has been chosen to about 0,25 after tests on offshore structures, but SSABs handbook [7] indicate the following:

$a \approx 0{,}15$ fillet welds
$a \approx 0{,}10$ butt welds, double V welds.

For thicknesses below 15 mm, for unwelded material and spot welds it is always conservative to put $j_t = 1$. There is also a table in the “Design and fabrication in high strength sheet steel” [7] which extrapolates the formula below t_0 = 15 mm down to as small t values as 1 - 4 mm.

4 Testing and material

4.1 Test specimen

In this chapter facts about the test specimens and fatigue testing will be dealt with.

4.1.1 Material

The material used for the fatigue testing is the SSAB Tunnplåt steel DOMEX 550 MC, which is an extra high strength hot rolled steel with the minimum yield strength 550 MPa and the tensile strength minimum 600 MPa and maximum 760 MPa, respectively. The chemical composition is presented in Table 2.

Alloy	C	Mn	Si	P	S	Al	Nb	V	Ti
[%]	0,12	1,80	0,30	0,025	0,010	0,015	0,09 max.	0,20 max.	0,15 max.

Table 2 The chemical composition of DOMEX 550 MC.

4.1.2 Description of test specimens

The specimens, of the type non-load carrying transverse fillet weld (cruciform) were made in three different thicknesses 3, 6 and 12 mm. Within this thesis work 6 mm and 12 mm were tested.

Figure 3 shows a picture of a 12 mm cruciform specimen. The specimens were scaled to have the same geometrical qualifications Table 3, the length L varied between the specimens; this variation is not expected to have an effect on the test results.

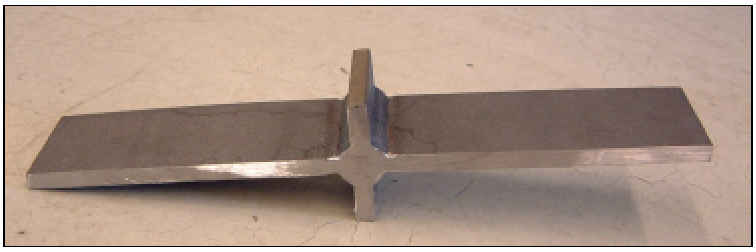

Figure 3 Picture of a cruciform joint t = 12 mm.

Thickness [mm]	Specimen width [mm]	Stiffener length [mm]	Effective Throat [mm]
$t = 3$	$7 \cdot t = 21$	$3 \cdot t = 9$	$a_e = 2$
$t = 6$	$7 \cdot t = 42$	$3 \cdot t = 18$	$a_e = 3$
$t = 12$	$7 \cdot t = 84$	$3 \cdot t = 36$	$a_e = 6$

Table 3 Geometrical conditions for the specimens.

Welding

The four welds are welded in the order shown in Figure 4 to avoid to big deformations because of the welding. The welds are created with MAG (Metal Active Gas) with shielding gas MISON 8 and the filler metal OK12.51. Before the welding the specimens were shot blasted to avoid cold laps. Cold laps are surface going flaws in the fillet weld toe with the depth of between 30 to 200 µm. The imperfections descends from an over run by the weld pool on the surface ground material with insufficient fusion as a result [12].

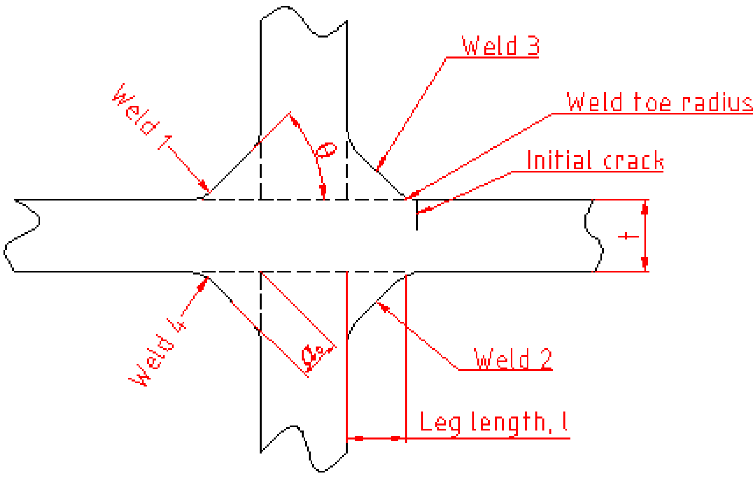

Figure 4 Weld sequence and parameters.

The 12 mm specimens were welded at Volvo and the 6 mm at SSAB. This might cause a difference in weld qualities, and the 12 mm seemed to have a more even quality than the 6 mm's. Below in Figure 5 a weld at a 12 mm specimen is shown.

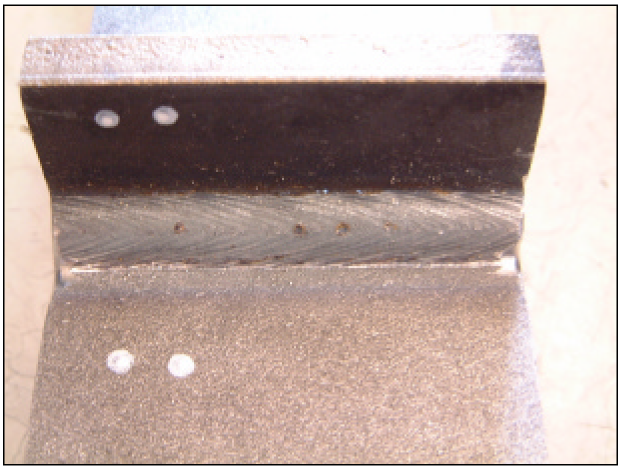

Figure 5 Weld at 12 mm specimen.

4.1.3 Geometry parameters

Effective throat
The effective throat thickness a_e, was set to be 2, 3 and 6 mm,respectively for the 3, 6 and 12 mm specimens. Measurement of the 12 mm specimens were the throat thickness is supposed to be 6 mm shows that the effective throat thickness varies between 6 and 7 mm with an average of 6,5 mm.

Weld toe radius
The weld toe radius has influence of the stress concentration and therefore it's interesting to measure it. The original aim with the measurements was to on an individual specimen

level locate the crack initiating point and measure the initial crack and weld toe radius there. But unfortunately no initial points could be found. So the weld toe radii were just measured at random points along the welds on the specimens.

To measure the radii, a string of silicone was applied at the toes before the testing. After the testing slices were cut out of the silicone strings and photographed in a microscope. The pictures were processed in a computer program were a circle were fitted at the weld toe, Figure 6. After calibration the program calculated the radius.

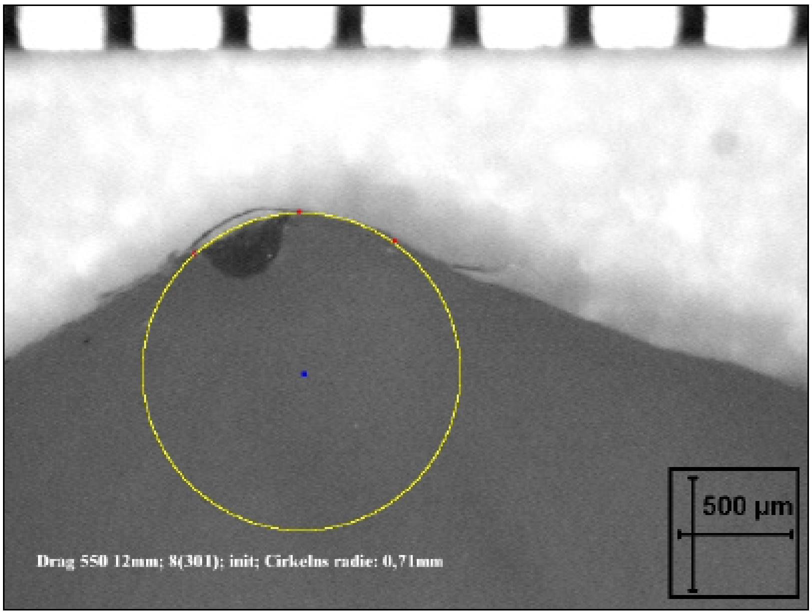

Figure 6 Estimation of weld toe radius.

It is not an easy task to fit the circle at the weld toe. From Figure 6 it’s obvious were the circle fits but in many of the slices it was hard to decide if a local or global radius should be measured. In Figure 7 the local radius (the circle in the right picture) were estimated to 0,22 mm but a circle with a much bigger radius could also be fitted. In this case the bigger circle has the radius 1,1 mm. This is an extreme case chosen to illustrate the difficulties with measuring the weld toe radius. Probably the local toe radius has more influence on the crack initiation than the bigger global.

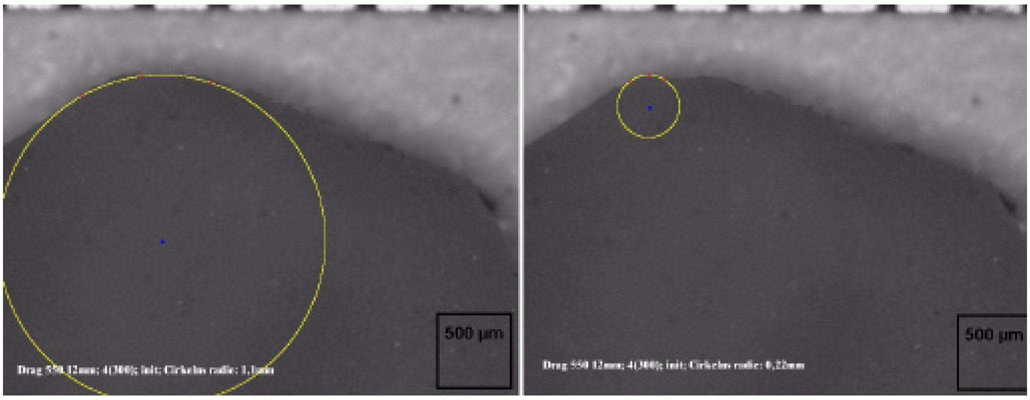

Figure 7 A circle with larger radius could also be fitted.

Approximately sixty measurements were made and the weld toe radius fluctuated between 0,15 and 2 mm. For the 12 mm specimens the radius had an average of 0,62 mm and for the 6 mm thick specimens the average radius was 0,78 mm. This difference indicates that the radii might be smaller for the 12 mm specimens but due to uncertainties in the measurements no conclusions can be withdrawn.

Weld angle
The weld angle (q), see Figure 4, were measured on the 12 mm specimens and also here the quality was very even and the angle varied only with a couple degrees, but since the inaccuracy in the measuring was bigger than that, the weld angle is approximated to 45°.

Leg length
Due to the variation of throat thickness the leg length l, see Figure 4, varies between 8,5 and 9,9 mm on the 12 mm thick specimens. According to Örjasäter [1] the fatigue strength increases with leg length. But how much influence the leg length has in this case is difficult to say.

Initial flaw size
As mentioned before no initial points or cracks could be found on the specimens, but in literature the initial crack a_0 is usually measured or approximated to 0,1-0,2 mm for welds.

For example British standard BS 7910:1999 [13] recommend the initial flaw size a_i between 0,1 and 0,25 mm.

4.2 Fatigue testing

4.2.1 Execution

The fatigue specimen were tested with constant amplitude with the stress ratio R = 0,01 in bending stress and R = 0 in tension stress with the frequency 10 Hz in both cases. The testing was performed at SSAB Tunnplåts laboratory in Borlänge.

One specimen in each test series was instrumented with strain gauges, to check the accuracy of the loading and the general arrangement.

4.2.2 Tension

Only 12 mm were tested in tension during this thesis work, so it is not possible to draw any conclusions about the thickness effect from this testing. The results are presented in Figure 10.

The results were compared with test results from Maddox [14] where 13 mm non-load carrying cruciform joints made of a BS 4360 50 B steel with yield strength 398 MPa with 10 mm thick attachment loaded in tension were used.

The fatigue life results from Maddox is a little bit lower than the fatigue life results in this study, e.g. at 200 MPa fatigue life were $0{,}2 \cdot 10^6$ cycles according to Maddox. In this study the fatigue life at 200 MPa were $0{,}3 \cdot 10^6$ cycles. Possible explanations for the difference in fatigue life are difference in weld quality, attachment thickness and different material.

4.2.3 Bending

The test specimens also were tested in three point bending Figure 8. Maybe four point bending would have given a more accurate result, four point bending gives a even moment over the welds and attachments and effects from the fact that the attachments are not straight receives a lesser signification. The results are shown in Figure 10.

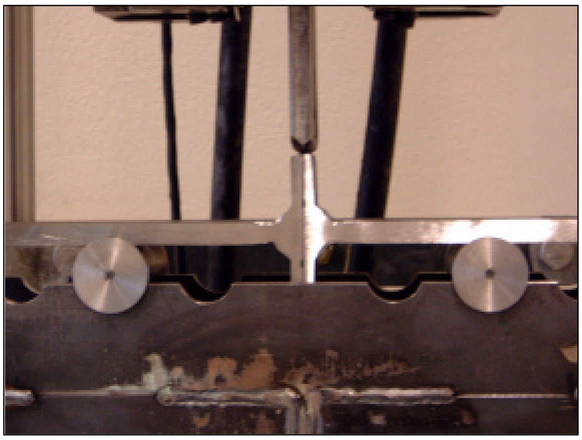

Figure 8 Three point bending of 12 mm sample.

4.2.4 Test results

Figure 9a shows a 12 mm specimen’s fracture surface after it has been fatigue tested and afterwards broken up. It is not possible to tell where the initial crack started and the initial crack depth. Probably there were several small cracks as illustrated by the red cracks in Figure 9b. When the cracks grow in the direction of the arrows in Figure 9b they will unite in one large semi-elliptical crack.

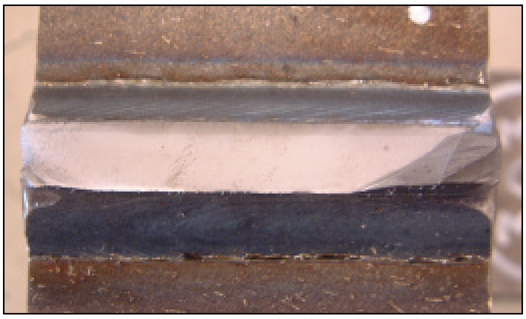

Figure 9a A specimens fracture surface.

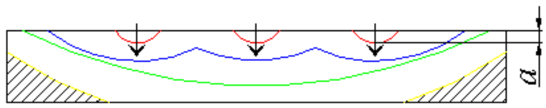

Figure 9b Plausible crack growth scenario.

The outcome of the fatigue testing is shown in the fatigue life diagram, Figure 10.

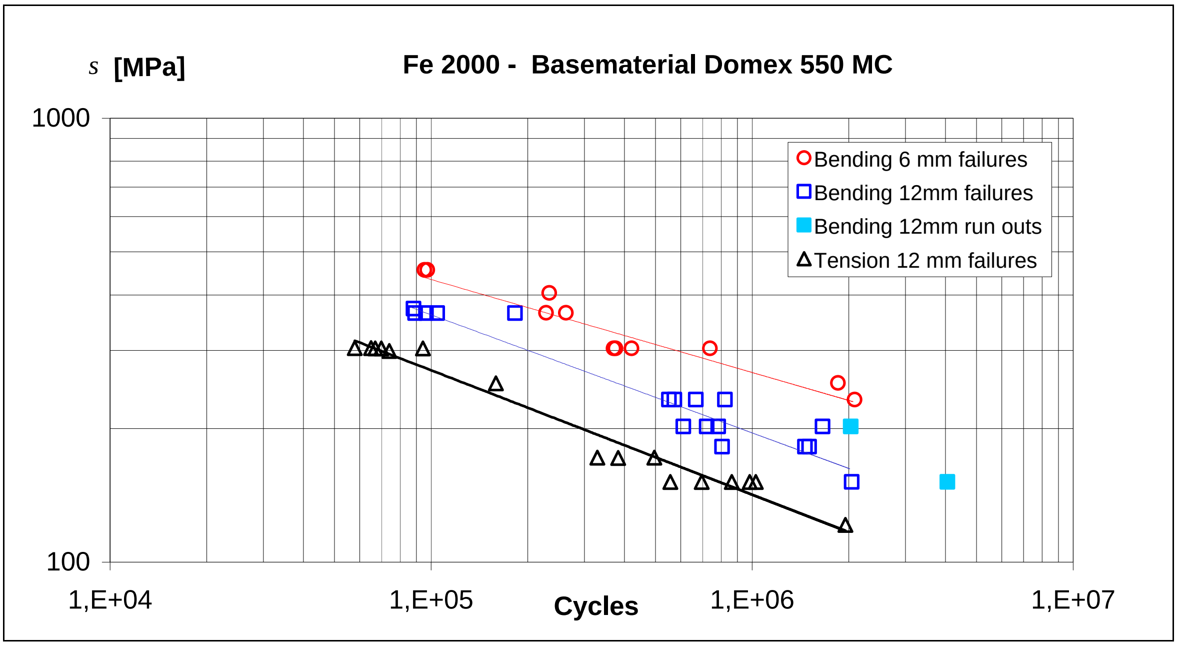

Figure 10 Fatigue life diagram for 6 and 12 mm specimens.

The slopes of the curves are different. For welded steels the slope is usually about *k* = - 3. In this case the slopes are *k* = - 3,6 for 12 mm specimens loaded in tension, *k* = - 3,8 for 12 mm specimens loaded in bending and *k* = - 4,78 for 6 mm specimens loaded in bending.

In order to compare the results for 6 and 12 mm specimens loaded in bending, the slopes are forced to *k* = - 3 according to IIW recommendations [8], Figure 11.

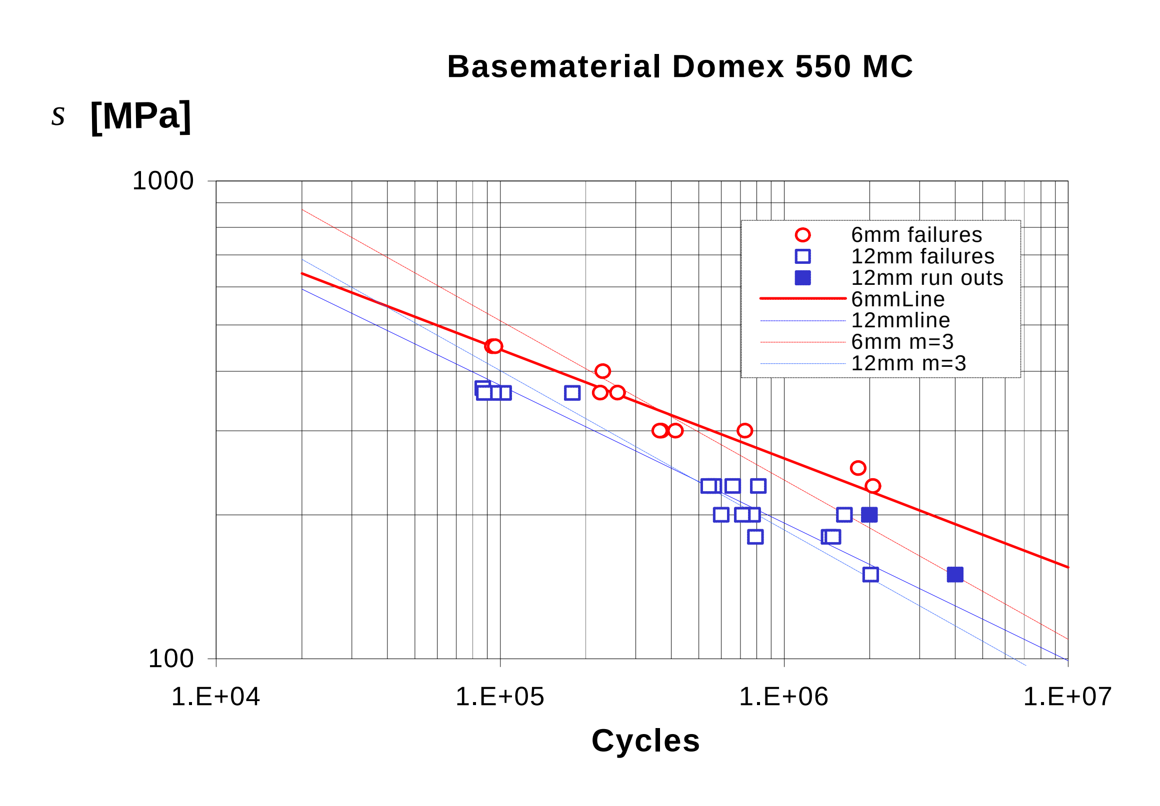

Figure 11 Specimens of 6 and 12 mm bending results forced to slope *k* = - 3.

4.2.5 Thickness effect exponent obtained from the test results

According to the test results the fatigue life is approximately twice as long for the 6 mm specimens than for the 12 mm specimens.

It is also possible to make an estimation of the thickness correction exponent n in the formula for the thickness effect (1), the reference thickness t_0 is to be found in many norms, standards and articles and varies between 15 - 32 mm.

The values for s were picked from Figure 11 with N = 200000 cycles.

- Bending 6 mm s_{b6} = 400 MPa
- Bending 12 mm s_{b12} = 320 MPa

The n factor for bending is calculated as shown below using formula (1). The size of the t_0 factor doesn't change the results.

Bending:

$$t = 12 \text{ mm} \Rightarrow s = s_0 \cdot \left(\frac{t_0}{12}\right)^n = 320 \Rightarrow s = s_0 = 320 \text{ MPa}$$

$$t = 6 \text{ mm} \Rightarrow s = 320 \cdot \left(\frac{t_0}{6}\right)^n = 400 \Rightarrow n = 0{,}32$$

In bending the n factor is approximated to n = 0,322.

5 Fracture Mechanics

To calculate the geometric factor *f* in formula (3) and the crack growth with Paris law (4) the stress intensity factors K_I has to be evaluated. This was numerically done by use of the finite element method (FEM).

5.1 FEM

To numerically calculate the stress intensity factors for the cracks in the specimens, two-dimensional (2D) models of the specimens were made in a FEM program.

For FEM calculation SSAB Tunnplåt uses a pre-processor called Hypermesh and ABAQUS for the solving and post-processor process. The numerical analyses were carried out on an IBM RS 6000 workstation. Young's modulus, E, was equal to 210 GPa and Poisson's ratio, ν, was equal to 0,3.

5.1.1 Modelling of the crack tip region

The element type chosen for the 2-D analyses was the plane strain biquadratic 8-noded CPE8 solid element and the plane strain quadratic 6-noded CPE6 solid element.

To model the crack tip the 8-noded isoparametric elements were collapsed to a triangular element where the a, b, c nodes has the same geometric location as the crack tip, the midside nodes on the sides are located ¼ of the side length from the crack tip, see Figure 12 [15].

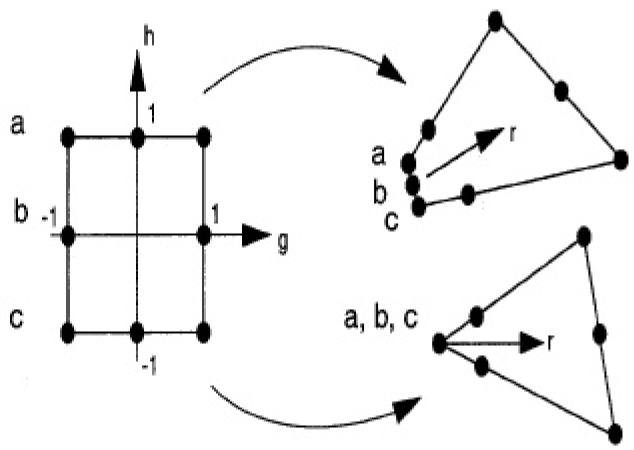

Figure 12 Collapsed two-dimensional element [15].

In the finite element model contour integral evaluations along the crack front is possible. A contour (domain) is a ring of elements surrounding the crack tip or crack front from one crack face to the opposite crack face.

Abaqus automatically finds the elements that form each ring from the node sets given as the crack tip or crack front definition. The user must specify the number of contours to be used. Abaqus calculates the contour integrals and from which the *J*-integral and the stress intensity factor, K_I, are extracted.

The *J*-integral is widely accepted as a fracture mechanics parameter for both linear and non-linear material response. It is related to the energy release associated with crack growth and is a measure of the load intensity at a notch or crack tip. If the material response is linear, it can be related to the stress intensity factors K_I, K_{II} and K_{III}. K_{I-III} characterizes the influence of load or deformation on the magnitude of the crack-tip stress and strain fields and is a measure of the propensity for crack propagation i.e. the crack driving forces. This feature, to relate the *J*-integral to *K*-factors is implemented in ABAQUS [16].

The elements surrounding the crack tip in the first contour is the collapsed 8-node elements and in the second contour the ordinary 8-node elements are used. In fig. 13 a typical model with two contours is shown. [16]

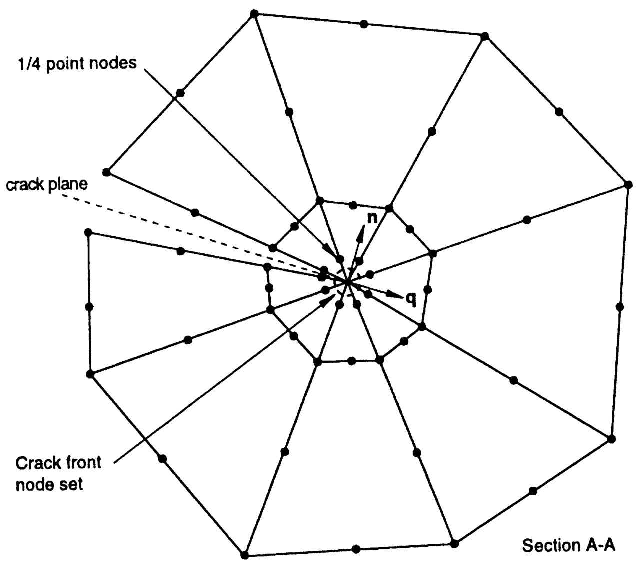

Figure 13 Showing the collapsed elements surrounding the crack tip and the two contours [15].

There are three methods in dealing with the singularity in the crack tip [16].

1. Create a square root singularity. Constrain node a-c to move together by tie them together or give them the same node number this option is most suitable for linear elasticity.
2. Create a 1/r singularity. Let node a-c be free to move independently and let the midside node remain at the middle of the element. This singularity is correct for the perfectly plastic case.
3. Create a combined square root and 1/r singularity. If the a-c nodes are free to move independently and the midside nodes are moved to the ¼ points. This combination is usually best for a power-law hardening material.

5.1.2 The FEM model used for the analysis of the cruciform joint

Two specimens with the geometry obtained from the real specimens were made. The two models were scaled to each other. The only thing that's not scaled was the weld toe radius and initial crack size, a_0. They were constant for the both thicknesses. The radius was set to the average of the measured radius at the specimens, 0,6 mm, and the initial crack was set to, 0,1 mm.

In the models, cracks with different depths originated from the weld toe were modelled, about six per specimen. The initial crack a_0 were for the both thicknesses set to 0,1 mm and the end crack to a_f = 4 mm for the 6 mm specimen and 8 mm for the 12 mm thick specimen. Figure 14 shows a complete 2D FEM mesh of a 6 mm specimen.

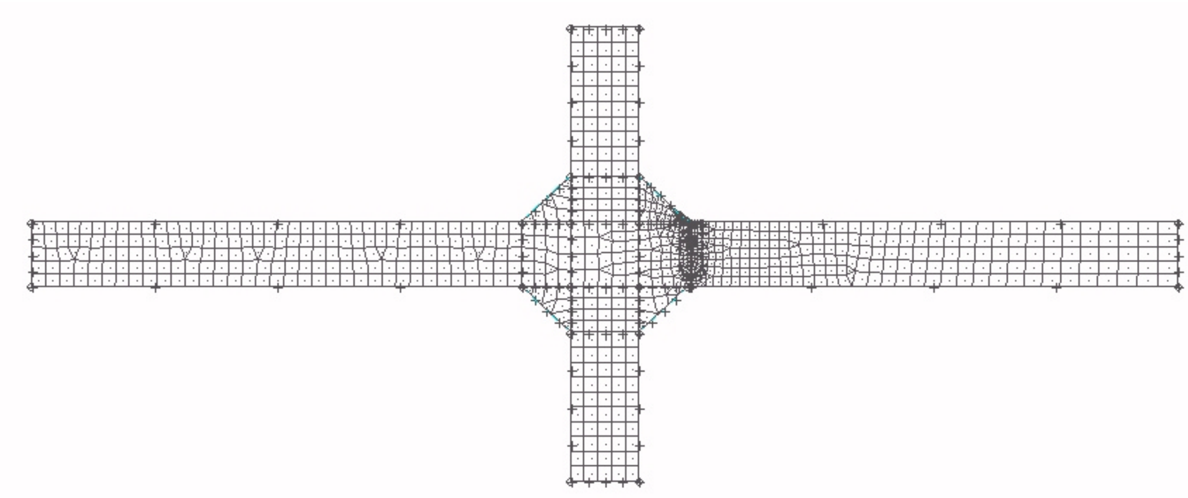

Figure 14 The specimen modelled in Fem.

It is recommended to have an angle between 10° (accurate result) and 22,5° (moderately accurate result) in the elements at the crack tip, in the Abaqus manual. In this analysis 16 and 18 elements around the crack tip was used, resulting in the angles 22,5° for the 12 mm thick specimens and 20° for the 6 mm thick specimens, respectively. In Figure 15 the mesh around an unloaded crack is shown.

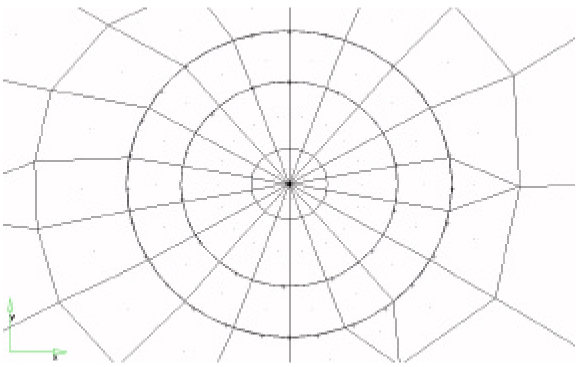

Figure 15 Shows the mesh structure at the crack tip.

To deal with the singularity at the crack tip, method three in chapter 5.1.1 a combined square root and 1/r singularity was used for the FEM calculations. A couple of FEM calculations using a square root singularity (method one in chapter 5.1.1) were also performed for a comparison. The results indicate approximately 1 % difference between the stress intensity factors calculated using the two different methods.

The deformed meshes for the load cases tension and bending are shown in Figure 16 and 17, respectively.

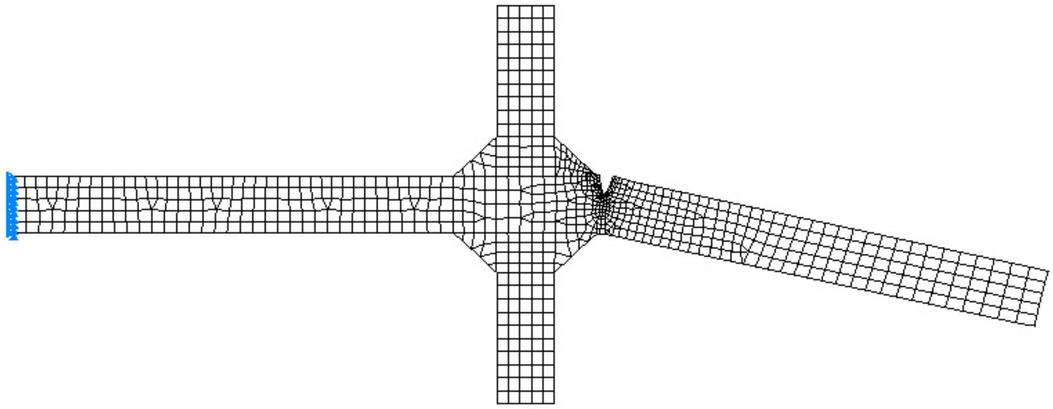

Figure 16 Model with a/t = 0,33 mm loaded in tension.

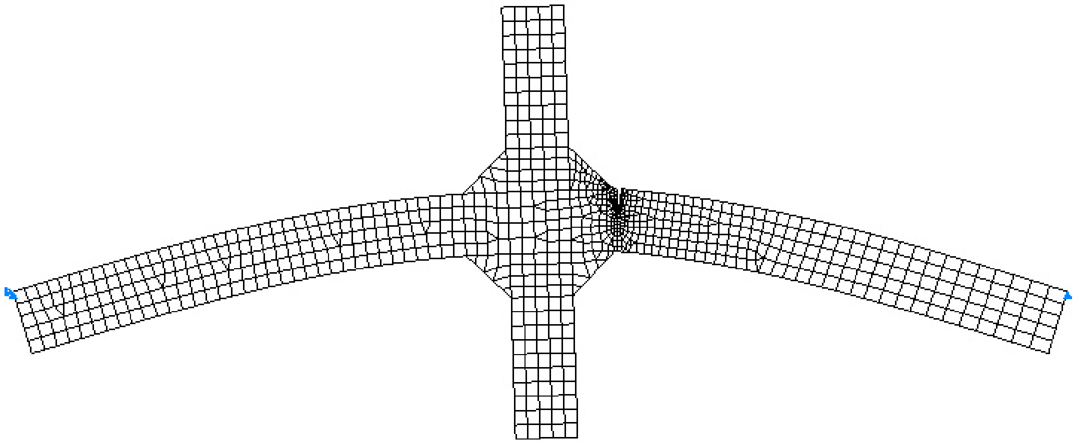

Figure 17 Model with a/t = 0,33 mm loaded in bending.

The 2D models of the sectional area of the cruciform joint contained the weld toe radius at the weld where the crack simulation was made is shown in Figure 18.

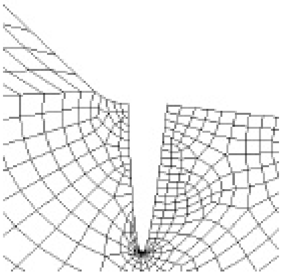

Figure 18 An open crack with origin at the weld toe.

Figure 19 shows the effective von Mises stress in the crack tip region (deformed mesh).

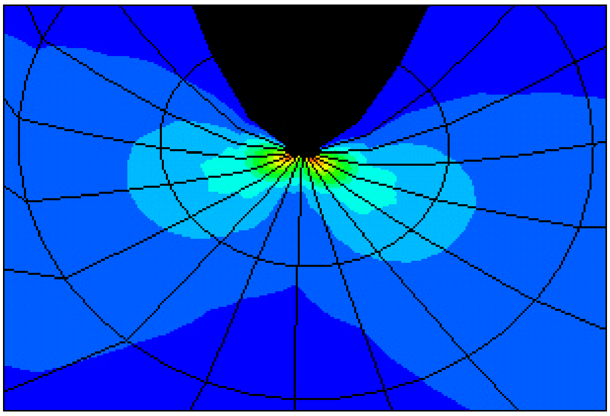

Figure 19 Crack tip zoomed in to visualize the von Mises stresses (deformed mesh).

To calculate the forces to be applied on the FEM model, common beam theory were used. The maximum stress was set to s_{max} = 1 MPa in the middle of the joint, both for tension and bending. This was calculated with common beam theory assuming the specimen had a rectangular cross section area and was uncracked.

The force to be applied to receive 1 MPa in tension was calculated with the formula

$$F = s \cdot A \ , \qquad (11)$$

where $A = w \cdot t$, $w =$ width and $t =$ thickness of the specimen.

In bending, where the applied stress (s) is the normal stress, equal to

$$s = \frac{M}{W} \ , \qquad (12)$$

Where M and W is,

$$M = \frac{F \cdot L}{4} \qquad (13)$$

$$W = \frac{w \cdot t^2}{6} \qquad (14)$$

5.1.3 Results

According to the Abaqus manual [16] strong variations in the *J*-integrals estimated from the different rings of elements (contours), commonly called domain or contour dependence, indicates a need for mesh refinement. The stress intensity factors (*K*) have the same domain dependence features as the *J*-integral.

The *J*-values obtained from contour 1 and contour 2 differed most for shallow cracks with up to 28% and at deeper cracks the difference was as small as 3 % for the 6 mm models. For the 12 mm thickness models, which contained a finer mesh around the crack, the results varied between 23% and 0%. Probably the use of more contours, up to ten instead of the two used here and a finer mesh should give more accurate result in the case with shallow cracks. A larger number of contours were not possible to attain with the current mesh design.

In the Abaqus manual [16] it is suggested that the second contour gives the most accurate result. So all calculations are based on the values obtained from the second contour. The *K*-values from the second contour also gives the most even results for the calculated fatigue life.

The *K*-values obtained from ABAQUS were increasing with increasing crack depth.

Thickness Effect

Once the K_I factors had been evaluated the geometrical function *f* was calculated from formula (3) this factor is in many articles e.g. [4] referred to as the non-dimensional *K* or shape factor, Y. Figure 20 is showing the shape factor *f* as a function of *a/t*. In Figure 20 there is a clear difference between 6 and 12 mm.

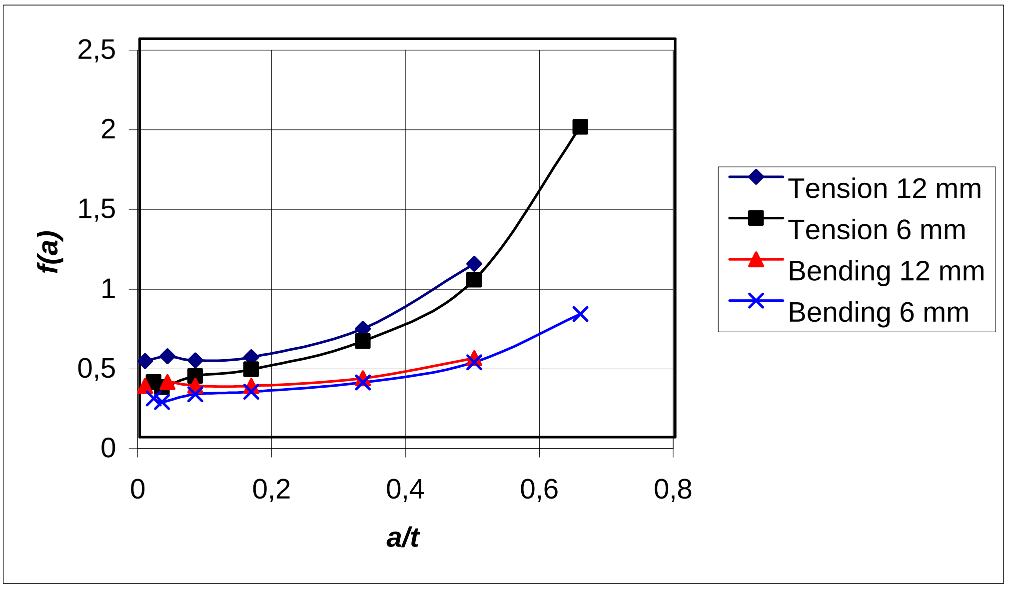

Figure 20 The shape factor as a function of the dimensionless *a/t*.

A curve fitting for the shape factors was performed with the following results:

$$f(a) = h_0 + h_1 \cdot \sqrt{\left(\frac{a}{t}\right)} + h_2 \cdot \left(\frac{a}{t}\right) + h_3 \cdot \left(\frac{a}{t}\right)^2 + h_4 \cdot \left(\frac{a}{t}\right)^2 , \qquad (15)$$

where the *h* factors is to be found in Table 4.

	h_0	h_1	h_2	h_3	h_4	Valid for *a/t*
12 mm thick specimens loaded in tension	0,409	1,057	-3,293	7,925	-3,222	0,008 to 0,5
12 mm thick specimens loaded in bending	0,268	0,811	-2,48	5,286	-3,415	0,008 to 0,5
6 mm thick specimens loaded in tension	0,489	-1,871	5,874	-11,995	15,093	0,02 to 0,67
6 mm thick specimens loaded in bending	0,261	-0,436	1,747	-3,871	4,882	0,02 to 0,67

Table 4 The factors used in formula 15.

Fatigue Life

With knowledge of the stress intensity factors K_I at different crack depths it was possible to make curve fits for $K_I(a)$ for the different thicknesses and loading and use Paris law (4) to calculate the expected fatigue life for the specimens, da/dN is the crack growth rate per cycle and C and m are material constants, but when the correct constants for Domex 550 MC was not available, the British Standard BS 7910 [13] recommendations with,

$$C = 5{,}21 \cdot 10^{-13}$$

and

$$m = 3$$

were used with units in [N] and [mm]. The initial crack a_0 was set to 0,15 mm in both 6 and 12 mm and the end crack a_f to 3 and 6 mm. The fatigue life is defined as the stage when the crack has reaches the final crack length a_f.

The stress intensity factors, *K,* were calculated in Abaqus for 1 MPa in the middle of the beam and were to be scaled to the different applied stresses. The formula to calculate the *K*-factor (2) and from that it's easy to realize that the *K* and *s* is linear scaled to each other. Therefore in tension the *K* factors were only multiplied with the applied stress, and in bending the stress were multiplied the same way but also scaled to be valid at the weld toes location at the beam and not in the middle.

In Appendix 1 a Mathcad code where both the procedures with making a function $K_I(a)$ and *f*(*a*) from the FEM results and the crack propagation fatigue life calculation with Paris law (4) is presented.

In Figure 21 the calculated fatigue lifes are compared with the results achieved from the fatigue testing in SSAB's laboratory. The red line represents the estimated values and the test results are represented by the blue boxes.

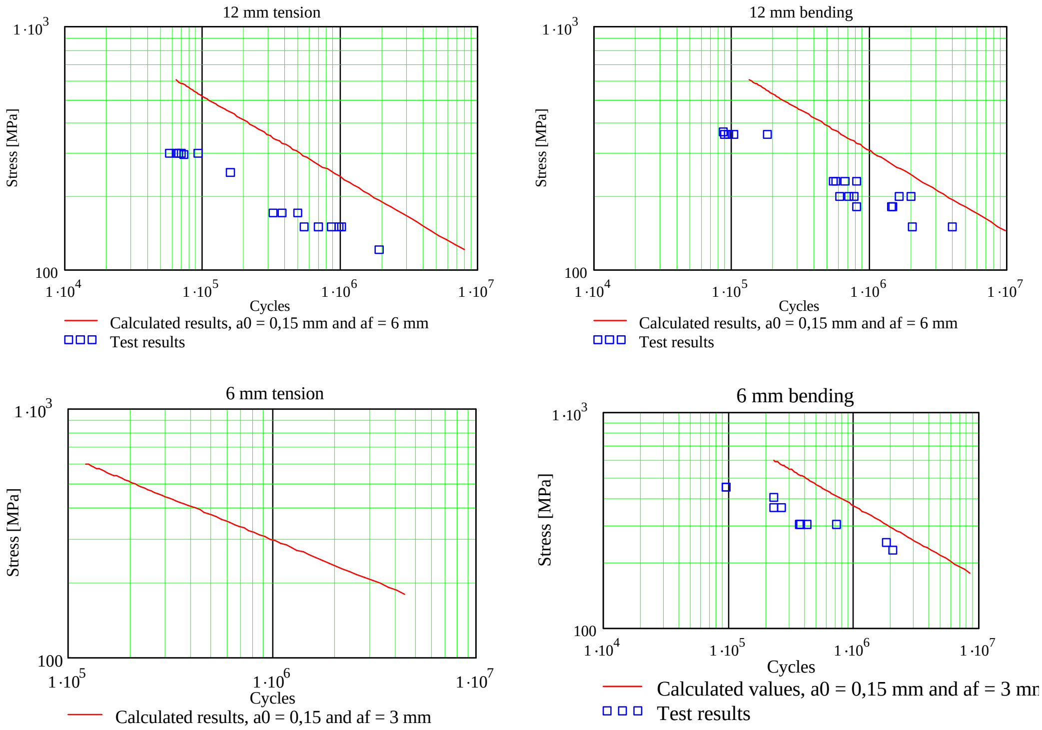

Figure 21 The results from the different fatigue calculations and the real test values.

It's also interesting to evaluate the stress level that will give the fatigue life at $2 \cdot 10^6$ cycles by using the FEM results. These results are given in Figure 22.

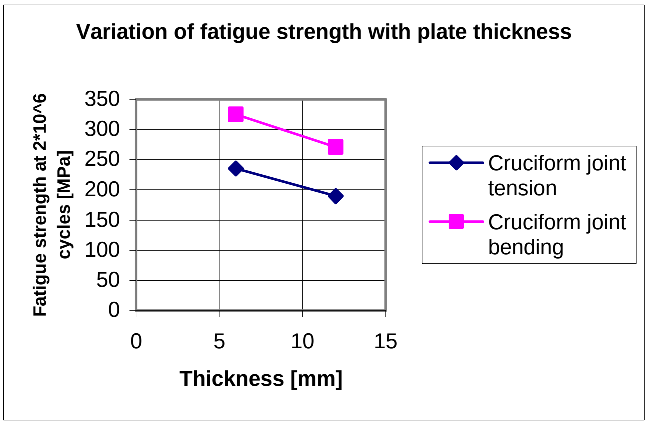

Figure 22 The difference in calculated fatigue strength between 6 and 12 mm.

There is quite a big difference between the calculated and test fatigue life. Possible explanations will be given in section, Conclusion and discussion, below.

To find more accurate material constants, *C* and *m*, the *C*-value was manipulated to get better accordance between the fatigue life length obtained with testing and the fatigue life length obtained with the numerical calculations. With the *C*-value set to $2{,}3 \cdot 10^{-12}$ and the *m*-value set to 3 a good accordance between the test results and the calculated result were obtained as shown in Figure 23.

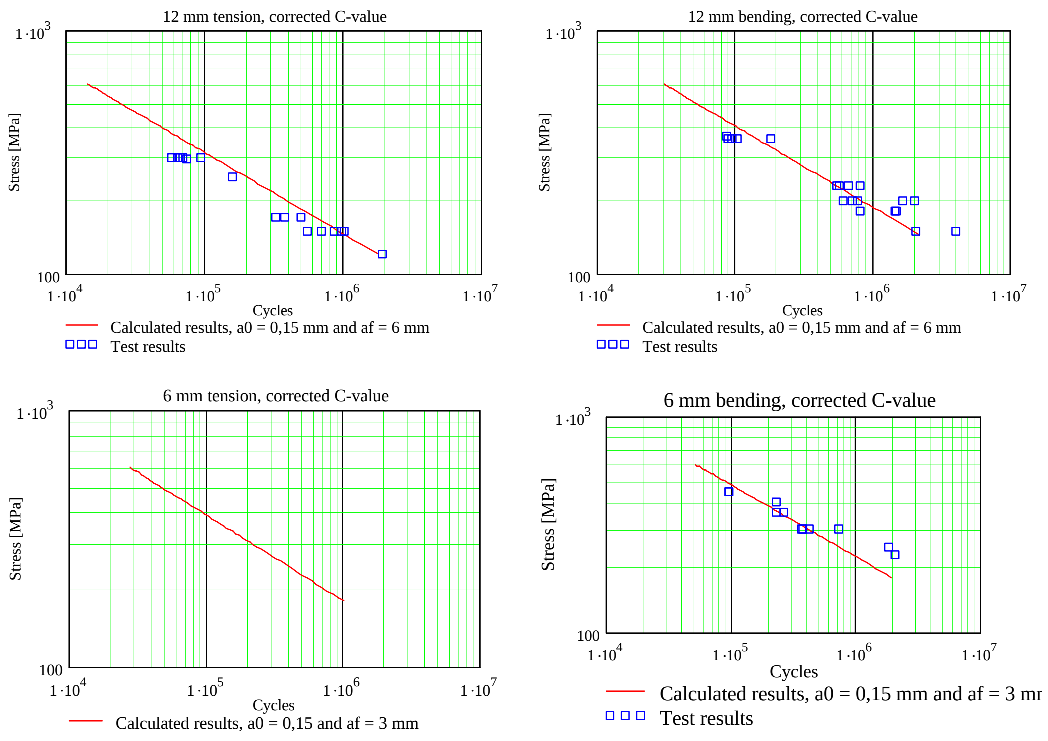

Figure 23 The results from the fatigue calculations, when the material constant, *C*, was equal to $2{,}3 \cdot 10^{-12}$ and the real test values.

Parametric study

To evaluate how big influence the size of the initial crack a_0 and the final crack a_f, respectively had on the calculated fatigue life, a parametric study, where the initial crack and the final crack values were changed, was performed. The results are presented in Table 5-8 and the constants in Paris Law was set to $C = 5{,}21 \cdot 10^{-13}$ and $m = 3$ according to BS [13] and it is obvious that the initial crack is by far more important than the end crack length. Table 5-8 is calculated for the same stress level, 300 MPa.

If the Tables are compared with the test results it is clear that to get the calculations to be in accordance with the real fatigue testing the initial crack, a_0, must be set to approximately 0,4 - 0,5 mm. One common assumption is to assume 0,2 mm initial crack length. Therefore 0,4 – 0,5 mm seems to be rather high

Init crack [mm]→	0,1	0,2	0,4	0,8
End crack				
1 mm	1,021	0,667	0,321	0,062
2 mm	1,162	0,808	0,463	0,020
4 mm	1,25	0,896	0,551	0,291

Table 5 Fatigue life / 10^6 for 12 mm bending at load 300 MPa.

Init crack [mm] →	0,1	0,2	0,4	0,8
End crack				
1 mm	2,495	1,186	0,451	0,078
2 mm	2,659	1,35	0,615	0,2418
4 mm	2,704	1,395	0,661	0,2873

Table 6 Fatigue life / 10^6 for 6 mm bending at load 300 MPa.

Init crack [mm] →	0,1	0,2	0,4	0,8
End crack				
1 mm	0,507	0,330	0,158	0,030
2 mm	0,572	0,329	0,224	0,095
4 mm	0,602	0,425	0,253	0,125

Table 7 Fatigue life / 10^6 for 12 mm tension at load 300 MPa.

Init crack [mm] →	0,1	0,2	0,4	0,8
End crack				
1 mm	1,577	0,583	0,199	0,037
2 mm	1,645	0,650	0,266	0,101
4 mm	1,656	0,662	0,277	0,112

Table 8 Fatigue life / 10^6 for 6 mm tension at load 300 MPa.

Estimation of the theoretical thickness effect exponent

It's also possible to make an estimation of the *n* factor in the formula for the thickness effect (1).

The s-values achieved for $N = 2 \cdot 10^6$ cycles is:

- Bending 6 mm s_{b6} = 325,2 MPa
- Bending 12 mm s_{b12} = 270,9 MPa
- Tension 6 mm s_{t6} = 235,2 MPa
- Tension 12 mm s_{t12} = 189,6 MPa

See Appendix 1 for explanation of how these values were achieved.
The *n* factor for bending and tension is calculated as shown below.

Bending:

$$t = 12 \text{ mm} \Rightarrow s = s_0 \cdot \left(\frac{t_0}{12}\right)^n = 270{,}9 \Rightarrow s = s_0 = 270{,}1 \text{ MPa}$$

$$t = 6 \text{ mm} \Rightarrow s = 270{,}9 \cdot \left(\frac{t_0}{6}\right)^n = 325{,}2 \Rightarrow n = 0{,}26$$

In bending the *n* factor is approximated to *n* = 0,26.

Tension:

$$t = 12 \text{ mm} \Rightarrow s = s_0 \cdot \left(\frac{t_0}{12}\right)^n = 189{,}6 \Rightarrow s = s_0 = 189{,}6 \text{ MPa}$$

$$t = 6 \text{ mm} \Rightarrow s = 189{,}6 \cdot \left(\frac{t_0}{6}\right)^n = 235{,}2 \Rightarrow n = 0{,}31$$

In tension the n factor is approximated to n = 0,31.

5.2 British standard

It is interesting to compare the test results with an accepted standard. Calculations made according to British Standard; Guide on methods for assessing the acceptability of flaws in metallic structures; BS 7910:1999 [13] is presented in Figure 24.

BS 7910:1999 shows how stress intensity factors can be calculated for several different cases. Annex M.5, which deals with welded joints will be used here.

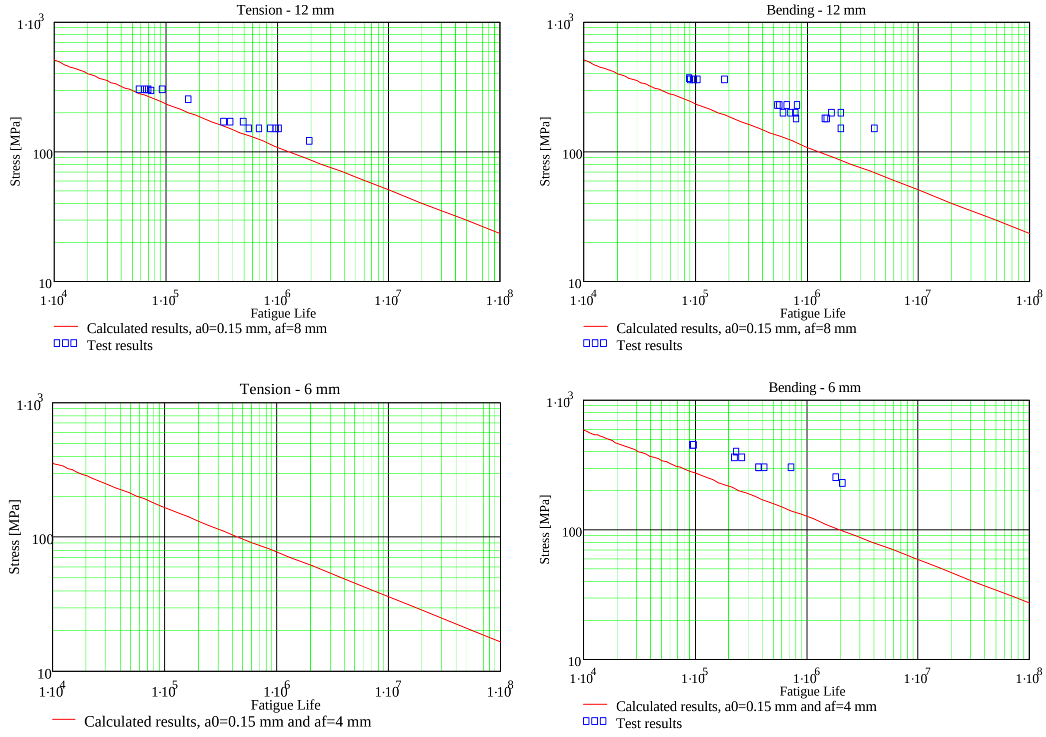

Figure 24 Calculations from BS 7910 compared with test results.

The British Standard doesn’t consider that the weld toe radius is constant, but when Paris Law (4) is used the fact that the initial crack often has the same length can be considered.

The stress intensity factors calculated with BS give much more conservative fatigue lengths than the values achieved from FEM.

It is not possible to se any thickness effect in the results from the BS calculations. In tension specimen of thickness 12 mm has longer fatigue life than specimens of thickness 6 mm and in bending specimens of thickness 6 mm has longer life than specimens of thickness 12 mm.

6 Discussion and conclusions

In literature three different reasons for the thickness effect are mentioned, the statistical size effect, the technological size effect and the stress gradient effect. However, this study shows that the effect can be explained directly by use of fracture mechanics analysis using the well-known fatigue growth law of Paris (4). In fact by use of the fracture mechanics concept a rather simple explanation can be found if self-similarity aspects are utilized. Consider for the moment a case where the toe radius does not influence the stress intensity factor solution. Furthermore, assume that the ratio of initial crack size over plate thickness is independent of the thickness of the plate, and that fatigue failure occurs at some ratio *a/t* close to unity. These assumptions lead to the following result: The stress intensity factors are cast in the forms

$K_I = s_0 \cdot \sqrt{p \cdot a_1} \cdot f_1$, $K_I = s_0 \cdot \sqrt{p \cdot a_2} \cdot f_2$, where $f_1 = f(a_1/t_1)$, $f_2 = f(a_2/t_2)$ and $t_1 > t_2$.

Thus, plate no. 2 refers to the thinner plate. Insertion into Paris law with $\Delta K_I = K_I$, as in the present study, yields the following crack growth rates

$$\frac{\mathrm{d}a}{\mathrm{d}N}\Big|_1 = C \cdot \left(s_0 \cdot \sqrt{p \cdot a_1} \cdot f_1\right)^m, \quad \frac{\mathrm{d}a}{\mathrm{d}N}\Big|_2 = C \cdot \left(s_0 \cdot \sqrt{p \cdot a_2} \cdot f_2\right)^m .$$

Now, compare the growth rates for the different plates at the same amplitude of applied stress, i.e. $s_0\big|_1 = s_0\big|_2$, and use self-similarity between the different plates, i.e. $f_1 = f_2$ during growth from the initial crack size to failure. The relationship between the growth rates then becomes

$$\frac{\mathrm{d}a}{\mathrm{d}N}\Big|_2 = \frac{\mathrm{d}a}{\mathrm{d}N}\Big|_1 \cdot \left(\frac{a_2}{a_1}\right)^{m/2} .$$

Thus, by this simple analysis it is obvious that the fatigue growth rate is smaller in the thin plate by the factor $(a_2/a_1)^{m/2}$. For the example studied in this investigation $a_2/a_1 = 0{,}50$ and $m = 3$, which gives a growth rate of the 6 mm thick plate equal to 0,35 times the growth rate of the 12 mm thick plate. In the experiments, as well as in the fracture mechanics analysis a factor of about 0,5 was found. The discrepancies can be explained by the fact that the initial crack sizes in fracture mechanics analyses had the same physical size and thus $a_{01}/t_1 \neq a_{02}/t_2$. Also, the weld toe radius is expected to influence the value of the geometric function *f*, as the finite element analyses (FEA) show in section 5 above. For shallow cracks $K_1\big|_{12mm} = 1{,}8 \cdot K_1\big|_{6mm}$ and for deep cracks $K_1\big|_{12mm} = 1{,}4 \cdot K_1\big|_{6mm}$. The difference between deep and shallow cracks is explained by the greater influence of the weld toe radius for shallow cracks.

The most common way to deal with the thickness effect is to use a correction factor that is calculated with the formula $(t_0/t)^n$. The correction factor is multiplied with the fatigue strength to achieve the reduced fatigue strength.

The reference thickness t_0 is a subject of discussion; it usually varies between 15 - 32 mm in the different standards, recommendations and articles. The exponent n depends of the type of joint and typically varies between 0,07 and 0,3. In Table 9 t_0 and n for different standards, recommendations and articles are listed. If $t < t_0$ it is usually recommended to choose $t = t_0$. No standards or recommendations consider the fact that there might be an increase in fatigue life for decreasing thickness below the limit value t_0.

	Gurney [6]	SSAB [7]	Eurocode [10]	British Standard [11]	BSK 99 [9]	IIW [8]
t_0 [mm]	22	15	25	16	25	25
n, depending of weld type	0,25	0,25; 0,15; 0,1	0,25	0,25	0,0763	0,1; 0,2; 0,3

Table 9 Different t_0 and n values in different standards, recommendations and articles.

The calculations and testing in this report has shown that the fatigue life for 6 mm non-load carrying joints is at least twice as long than for the 12 mm joints. This indicates that there is a real fatigue life improvement with decreasing thicknesses under the limit value t_0.

In fatigue testing it is always difficult to attain an even quality for test specimens. In this case different thicknesses were used and that caused different problems with e.g. deformations due to the welding process. The non-load carrying attachments probably caused some disturbance of the results in bending since the attachments are not straight and maybe not directly in contact with the big plate. Using four points bending instead of the three point bending could have improved the situation.

The FEM models were modelled without the narrow gap that is possible between the attachment and the main plate. But a rough estimation on another FEM model of a non-load carrying transverse fillet welded cruciform joint indicates that it doesn't make any big difference in tension and bending.

The n factor achieved from the testing and fracture mechanics calculations are presented in Table 10. The n values achieved from the fracture mechanics analysis probably are correct even though the fatigue life differed, because the ratio between fatigue life between 6 and 12 mm were approximately the same in the theoretical fatigue calculations and the fatigue testing

Test values bending	$n = 0{,}322$
Calculated values bending	$n = 0{,}26$
Calculated values tension	$n = 0{,}31$

Table 10 The t_0 and n factors achieved from testing and fracture mechanics calculation.

Table 11 shows the improvement of fatigue strength for an 8 mm plate with the formula $(t_0/\ t)^n$. T R Gurney [6] has found that the British Standard BS 7608 [11] can be extrapolated to thicknesses as low as 2 mm in some cases, therefore BS [11] is used here; see the literature survey for more information.

	Test results bending with t_0 = 12 mm	Calculated values bending t_0 = 12 mm	Calculated values tension t_0 t_0 = 12 mm	Sheet steel handbook (Fillet weld)	T R Gurney with British Standard
t = 8 mm	1,14	1,11	1,13	1,099	1,189

Table 11 The fatigue strength improvement for *t* = 8 mm.

The test results indicates that the sheet steel handbook [7] is a little to conservative in their approximation and that T R Gurney's [6] extrapolation of the British Standard [11] is little over estimated but all values doesn't differ more than 10 %.

The limit value chosen in Table 11 is t_0 = 12 mm. But, another bigger value maybe t_0 = 25 mm could also be used with the same factor n, e.g. the 8 mm plate in tension (25/8)^0,26 = 1,35 i.e. around 16 % higher fatigue strength than if t_0 were chosen to 12 mm.

7 References

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8 Appendix 1

Fatigue life calculations for 12 mm tension

Constants:

According to BS 7910:1999 p. 47 C = 5.21*10^-13 and m = 3 for, da/dN in [mm/cycle] and K in [N/mm^3/2]

$C\_BS := 5.21 \cdot 10^{-13}$ $n\_BS := 3$

Calculation with cracklength in mm and KI in Nmm^-3/2

$a0 := 0.15$ $af := 6$ a0 = initial crack and af = end crack

KI(a[mm],KI[N/mm^2*mm^1/2]) $KI(a) := 0.241 + 0.711 \cdot a - 0.144 \cdot a^2 + 0.025 \cdot a^3$

In FEM-calculations sigma was = 1, multilply KI with sigma to achieve correct values of the stress concentration factors.

Calculations

$\sigma := 300$ KI=σ*(π*a)^(1/2)*f

$$N(a0, af) := \frac{1}{C\_BS} \left[\int_{a0}^{af} \frac{1}{[\sigma \cdot (KI(a))]^{n\_BS}} \, da \right]$$

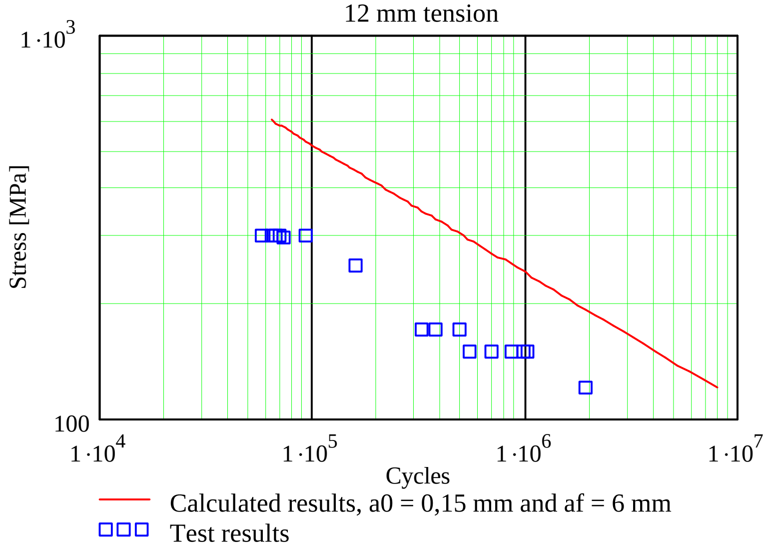
$N(a0, af) = 5.05 \times 10^5$

$i := 20..100$ $\sigma_i := \frac{i}{100} \cdot 600$

$$N1(a0, af, \sigma1) := \frac{1}{C\_BS} \left[\int_{a0}^{af} \frac{1}{[\sigma1 \cdot (KI(a))]^{n\_BS}} \, da \right]$$

$Ntest := Mtest^{\langle 0 \rangle}$ $\sigma test := Mtest^{\langle 1 \rangle}$

$$Mtest := \begin{pmatrix} 157000 & 250 \\ 73000 & 296 \\ 325000 & 170 \\ 377000 & 170 \\ 1011000 & 150 \\ 548000 & 150 \\ 57000 & 300 \\ 1923000 & 120 \\ 968000 & 150 \\ 687000 & 150 \\ 851000 & 150 \\ 69000 & 300 \\ 93000 & 300 \\ 489000 & 170 \\ 66000 & 300 \\ 64000 & 300 \end{pmatrix}$$



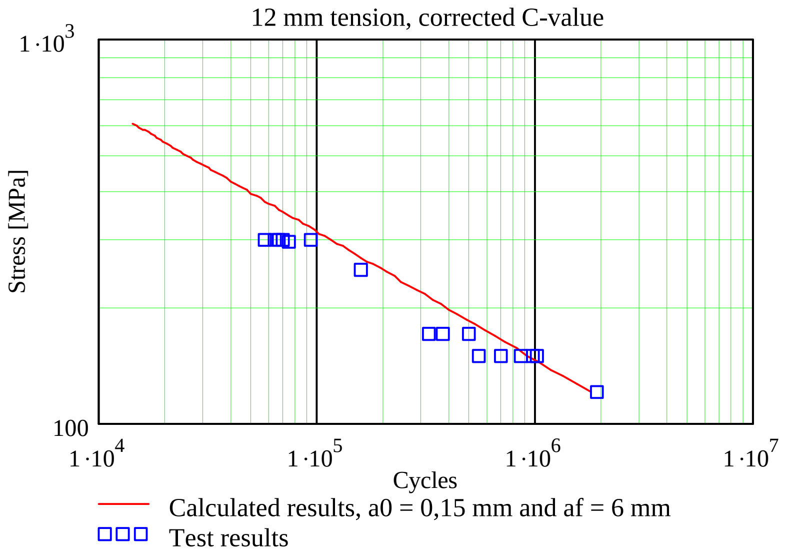
$$\sigma 2 := \frac{\left[\displaystyle\int_{a0}^{af} \frac{1}{(KI(a))^{n\_BS}}\, da\right]}{\displaystyle\int_{0}^{2\cdot 10^{6}} C\_BS\, dN}$$

Calculate stress σ3 in weld toe for the fatigue life 2*10^6 cycles n = 3 according to BS standard

$\sigma 3 := \sqrt[n\_BS]{\sigma 2}$ $\qquad \sigma 3 = 189.615$

With C = 2,3*10^-12:

$$N1(a0, af, \sigma 1) := \frac{1}{2.3\cdot 10^{-12}} \left[\int_{a0}^{af} \frac{1}{[\sigma 1\cdot(KI(a))]^{3}}\, da\right]$$



The curve fitting for KI(a), a[mm], KI [MPamm^1/2]

In the calculations x is crack depth and y is the stress intensity factors achieved from FEM calculations.

$x_0 := 0.1 \quad x_1 := 0.5 \quad x_2 := 1 \quad x_3 := 2 \quad x_4 := 4 \quad x_5 := 6$

$y_0 := \frac{8.35}{\sqrt{1000}} \quad y_1 := \frac{19.90}{\sqrt{1000}} \quad y_2 := \frac{26.72}{\sqrt{1000}} \quad y_3 := \frac{39.28}{\sqrt{1000}} \quad y_4 := \frac{75.36}{\sqrt{1000}} \quad y_5 := \frac{147.62}{\sqrt{1000}}$

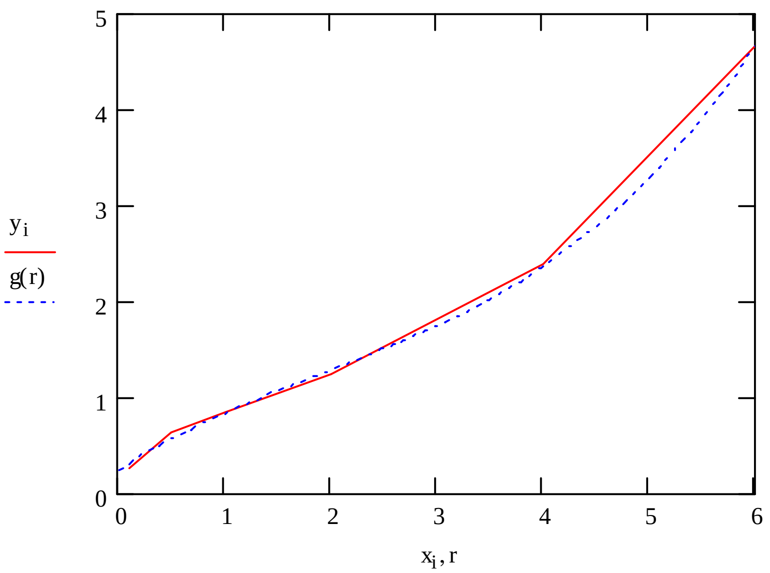
$$F(x) := \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \\ 0 \end{pmatrix}$$

$S := \text{linfit}(x, y, F) \qquad S = \begin{pmatrix} 0.241 \\ 0.711 \\ -0.144 \\ 0.025 \\ 0 \end{pmatrix}$

$r := 0, 0.1 .. 6$

$i := 0 .. 5$

$g(t) := F(t) \cdot S$



Curve fitting for f(a) as a function of a/t, for 12 mm tension. a in the calculations means the ratio a/t.

$a_0 := 0.00833$ $a_1 := 0.041667$ $a_2 := 0.0833$ $a_3 := 0.16667$ $a_4 := 0.333$ $a_5 := 0.5$

$f_0 := 0.4766$ $f_1 := 0.5081$ $f_2 := 0.4824$ $f_3 := 0.5014$ $f_4 := 0.6803$ $f_5 := 1.0880$

$$F(a) := \begin{pmatrix} 1 \\ \sqrt{a} \\ a \\ a^2 \\ a^3 \end{pmatrix}$$

$S := \text{linfit}(a, f, F)$ $\qquad S = \begin{pmatrix} 0.409 \\ 1.057 \\ -3.293 \\ 7.925 \\ -3.222 \end{pmatrix}$

$r := 0, 0.0083 .. 0.5$

$i := 0 .. 5$

$g(t) := F(t) \cdot S$