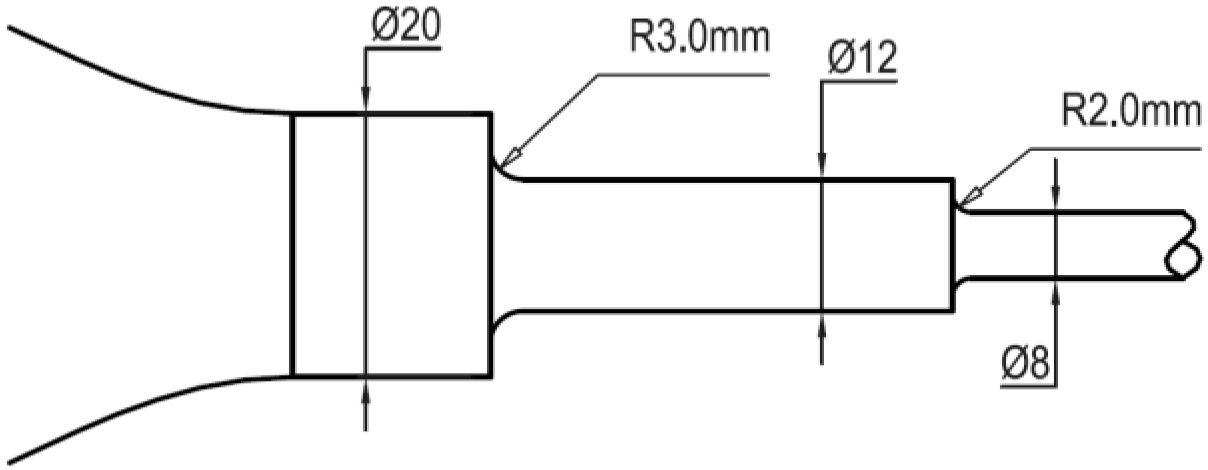
Test 1 Q2

The Units used as standard:
mm, N, MPa, sec, watts

N, mm, MPa, sec/min, watt/kW



A round and rotating shaft made from AISI 1050CD is part of a powered machine. During application, a tensile load of force from 10N to 500N and a twisting force from 5.0Nm to 14.0 Nm are needed. If the machine is designed to for infinite life determine the safety factor for the proposed design. Use the Modified Goodman for fatigue analysis.

It is obvious that the failure will occur at the shoulder of the 10mm shaft.

$F_{max} = 500\ N$ $\quad d = 8\ mm$ $\quad T_{max} = 14\ N\ m$

$F_{min} = 10\ N$ $\quad T_{min} = 5\ N\ m$

Material 1050CD steel. See Table A-18 $\quad S_{ut} = 690MPa$ $\quad S_y = 580MPa$

Endurance Limit $\quad S_{e'} = 0.5\ S_{ut} = 345\ MPa$

Surface Factor:CD/machined $\quad k_a = 4.51 S_{ut}^{-0.265} = 0.798$

For TWISTING LOAD: Round & rotating shaft $\quad d = 8\,mm$

If $2.79 \le d \le 51mm$ then Size Factor $\quad k_b = 1.24 d^{-0.107} = 0.993$

For AXIAL LOAD: $k_{c1} = 0.85$ $\quad$ *Temperature factor* $\quad k_d = 1$

For TWISTING LOAD: $k_{c2} = 1$ $\quad$ *Reliability factor* $\quad k_e = 1$

Load Factor $\quad k_c = k_{c1} \times k_{c2} = 0.85$ $\quad$ *Misc factor* $\quad k_f = 1$

Actual Endurance limit, $\quad S_e = k_a k_b k_c k_d k_e k_f S_{e'} = 232.23\,MPa$

Now to find out the loads and stresses applied on the shaft.

$d = 8\,mm$ $D = 12\,mm$ $r_f = 2\,mm$ $\frac{D}{d} = 1.5$ $\frac{r_f}{d} = 0.25$

Refer to A-13-7 pg 1004 Axial Load

$K_t = 1.5$

From Figure 6-20, pg 287

$S_{ut} = 690\,MPa$ & $r_f = 2\,mm$

===> $q = 0.82$

From equation 6-32 pg 287

$$K_f = 1 + q(K_t - 1) = 1.41$$

$$\sigma_{max} = K_f \frac{4F_{max}}{\pi d^2} = 14.026\,MPa$$

$$\sigma_{min} = K_f \frac{4F_{min}}{\pi d^2} = 0.281\,MPa$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 7.153\,MPa$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 6.873\,MPa$$

Refer to A-13-8 pg 1004 Torque

$K_{ts} = 1.19$

From Figure 6-21, pg 288

"Drawn" steel & $r_f = 2\,mm$

===> $q_s = 0.98$

From equation 6-32 pg 287

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1.186$$

$$\tau_{max} = K_{fs} \frac{16T_{max}}{\pi d^3} = 165.191\,MPa$$

$$\tau_{min} = K_{fs} \frac{16T_{min}}{\pi d^3} = 58.997\,MPa$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = 112.094\,MPa$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = 53.097\,MPa$$

$$\sigma'_m = \sqrt{\sigma_m^2 + 3\tau_m^2} = 194.284\,MPa$$

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = 92.223\,MPa$$

TEST FOR FIRST CYCLE YIELDING:

Langer's equation: $\frac{\sigma'_a}{S_y} + \frac{\sigma'_m}{S_y} = \frac{1}{\eta_y}$

$$\eta_y = \frac{S_y}{\sigma'_m + \sigma'_a} = 2.024$$

TEST FOR FATIGUE FAILURE :

Goodman $\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{\eta_f}$

$$\eta_f = \frac{1}{\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}} = 1.473$$

Few students may fail to see the term "drawn" in the chart and use the following values :

From Figure 6-21, pg 288, " " steel & $r_f = 2\,mm$ ===> $q_s = 0.94$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1.179$$

$$\tau_{max} = K_{fs}\frac{16T_{max}}{\pi d^3} = 164.133\,MPa$$

$$\tau_{min} = K_{fs}\frac{16T_{min}}{\pi d^3} = 58.619\,MPa$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = 111.376\,MPa$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = 52.757\,MPa$$

$$\sigma'_m = \sqrt{\sigma_m^{\;2} + 3\tau_m^{\;2}} = 193.041\,MPa$$

$$\sigma'_a = \sqrt{\sigma_a^{\;2} + 3\tau_a^{\;2}} = 91.636\,MPa$$

TEST FOR FIRST CYCLE YIELDING:

$$\eta_y = \frac{S_y}{\sigma'_m + \sigma'_a} = 2.037$$

TEST FOR FATIGUE FAILURE :

$$\eta_f = \frac{1}{\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}} = 1.483$$